Does the Price Gap Predict Inflation?: An Approach Based on the Theory of the Price Level

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Abstract

Despite the rapid expansion of liquidity in most countries since the 2000s, inflation has remained subdued for a considerable time. The Korean economy has also experienced a weak relation between money growth and inflation since the currency crisis. This raises the question of whether money growth leads to inflation with some time lag. This paper presents the possibility that the weak relation between the two is caused by a shift in the monetary policy regime, that is, from a monetary targeting regime to an inflation targeting regime.

In order to account for this, it is necessary to show how the price level is determined in the long-run depending on the monetary policy regime. This paper introduces a concept of the long-run equilibrium price level, and explains how the determination of the price level is affected by the policy regime. It turns out that the long-run equilibrium price level is determined by the money stock under the monetary targeting regime. As a result, the price gap, that is, the gap between the long-run equilibrium price level and the current price level, has a high predictability of future inflation. In contrast, under the inflation targeting regime, the long-run equilibrium price level is not determined by the money stock as money is supplied endogenously. Rather it is shown that the long-run equilibrium price level is pinned down by the interest rate. In this case, the price gap no longer predicts future inflation.

The results of this paper suggest that the relation between monetary developments and future inflation is more complex under the inflation

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targeting regime. Although money demand is stable over time, future portfolio adjustments can greatly affect the movements of the monetary aggregates and the price level.

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I. Introduction

The possibility of inflation induced by excessive liquidity has been repeatedly mentioned by many economists. The view that increasing liquidity in a financial markets would lead to future inflation is based on the opinion of monetarists who argue that prices are determined by the amount of money supply from a long-term perspective. The P-star model developed by Hallman, Porter and Small (1991) is also based on the view that the amount of money supply determines prices. This view is still prevalent in Europe. In the same context, the European Central Bank puts an emphasis on the movements of money supply in predicting inflation. Recently, Gerlach and Svensson (2003) also claim that the P-star model is really useful in forecasting inflation in Europe. Meanwhile, many central banks implement monetary policy focused on interest rates under inflation targeting, raising questions as to the relationship between money and prices. For Korea, along with the shift in its monetary policy after the Asian currency crisis, the relationship between money and prices has been changing. Thus, more systematic studies on these two variables are called for. In order to provide answers in these issues, it is necessary to identify which factors determine prices in the long-run. Notably, it is really critical to show how the determinants of prices may change in accordance with the policy regime adopted by central banks.

In this paper, I examine which factors determine prices in the long-run, mainly focusing on the theory of the price level or the theory of price level determination that has been systemized in academic fields. Many researchers have thought that there exists a price level determined in the long-run. This price level has been conceptualized as the long-run equilibrium price level. In economic theory, an equilibrium that emerges when all prices (including input prices) are adjusted in a flexible way is usually regarded as a long-run equilibrium. In this context, long-run equilibrium prices are the price level that appears in case all the prices in an economy are flexible.

The theory of price level determination is a field of macro economics in which economists study which factors determine nominal variables as well as the price level within an economy when all real variables are in equilibrium in the long-run. Classical economics maintains its stance of classical dichotomy, according to which it is possible to explain the process of determining the equilibrium of real variables and nominal variables within an economy separately. It is argued that the real variables are determined by supply factors such as the technology level while the nominal variables including the price level are determined by money supply. Meanwhile, there has been a widespread perception that it is more effective for central banks to control interest rates when the demand for money is volatile. However, some economists (Sargent and Wallace, 1975) who adopt the Rational Expectation Hypothesis have raised questions about the effectiveness of such monetary policy, asserting that, if central banks use interest rates as their policy instrument, then the price level could be undetermined.

Leeper (1991), Sims (1994), and Woodford (1994) tried more comprehensive and modern studies on the determinants of prices along with the development of the DSGE (dynamic stochastic general equilibrium) model. They think that prices are not determined by monetary factors alone, and suggest examples where prices are determined by money supply and cases where they are determined by fiscal policy measures. Their research has contributed to the development of the fiscal theory of the price level (FTP or FTPL), which puts importance on fiscal policy in determining the price level.

In contrast to the theory above, the monetary theory of the price level places its emphasis on monetary factors (money supply and the interest rate). The quantity theory of money and monetarism, the conventional view that money supply determines prices, are in the same category as the monetary theory of the price level.

In this study, I examine how the price level is determined in the long-run by using the monetary theory of the price level. It should be noted that the actual price level could exhibit some divergence from the long-run equilibrium price in the short-run due to the effect of price rigidity. If the price level converges to the long-run equilibrium price over time, price dynamics can be described as a complicated combination of determination of the long-run equilibrium price level and short-term adjustment process, in which, the actual price level converges to the long-run equilibrium price level. The process of the former can be verified by the theory of price level determination and the latter by the Phillips Curve theory. However, it is unclear whether prices would always converge to the long-run level regardless of the policy regime implemented by central banks. Discussions about this have gained little attention.

In this regard, I look into the long-run determinants of prices and additionally analyze whether the actual level could converge to the long-run equilibrium. Based on this, I examine how the variations in money stock affect the price movements triggered in accordance with the policy regimes of central banks. And from the results, I reinterpret the P-star model, which has attracted a great deal of attention as a model that predicts inflation, and assess its usefulness.[1)]

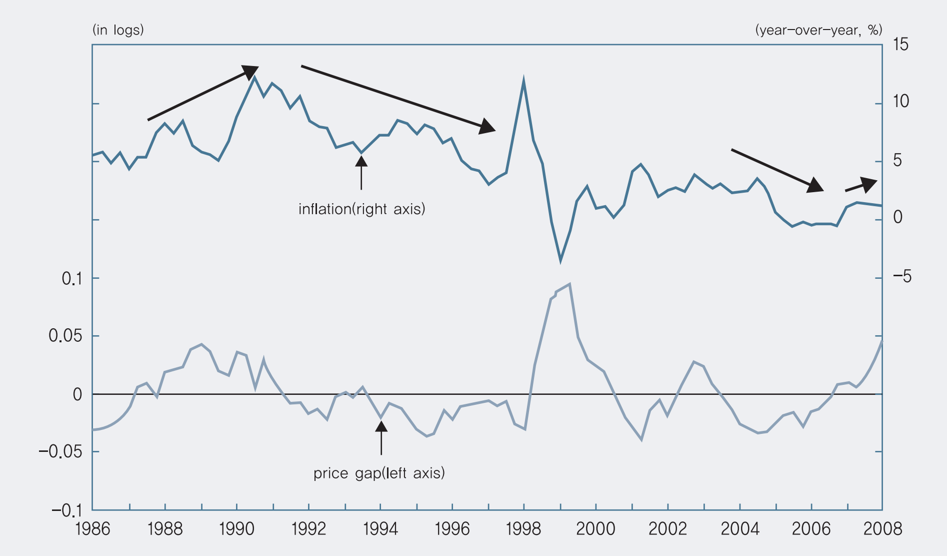
The remainder of this paper is organized as follows. In Section Ⅱ, I analyze how the shift in monetary policy has changed the relationship between money and prices. In Section Ⅲ, I investigate what factors determine prices in the long-run by using the monetary theory of the price level. In Section Ⅳ, I analyze how the actual price level exhibits different movements from that of long-run equilibrium prices and additionally analyze the predictability for inflation of the price gap which is a gap between two variables. Finally, I provide a summary of this study and set out some implications.

II. Shift in the Relationship between Money and Prices

The trend of money stock and prices in Korea shows that the growth rate of M2 (broad money) was around 20% before the eruption of the Asian currency crisis while the inflation rate was running at a high level. However, after the financial crisis, the inflation rate sharply decreased thanks to inflation targeting, along with the slowdown in the growth rate of money. To see how closely money and prices are linked, cross correlation between two variables can be examined. However, just with the simple comparison of the growth rate of money with the inflation rate, it is difficult to identify the causality between the two. The reason is that high inflation and real GDP growth rates increase the demand for money, leading to the faster growth rate of money. Even monetarists think that, only when money supply exceeds the demand for it, this generates inflation.

1) In the theory of price level determination that will be discussed in the next section, the equilibrium price level which emerges in case prices are flexible, $\widetilde{P}_t^*$ is defined as the long-run equilibrium price and is used to indicate this. In order to discern the long-run equilibrium price and P-star, $\widetilde{P}_t^*$ is used for indicating P-star

Figure 1 **Trend of price gap and inflation**



Note: The price gap is the difference between the log of P-star and the log of the actual GDP deflator. Inflation denotes the year-over-year change rate of the GDP deflator.

In this regard, the P-star model developed by Hallman, Porter and Small (1991) is more useful in figuring out the degree of excess supply of money than the level of money stock itself. In order to estimate P-star, which refers to the long-run equilibrium prices appearing at the level of current money stock, it is essential to grasp the long-run velocity and long-run equilibrium real GDP(or potential output) in the P-star identity equation below.

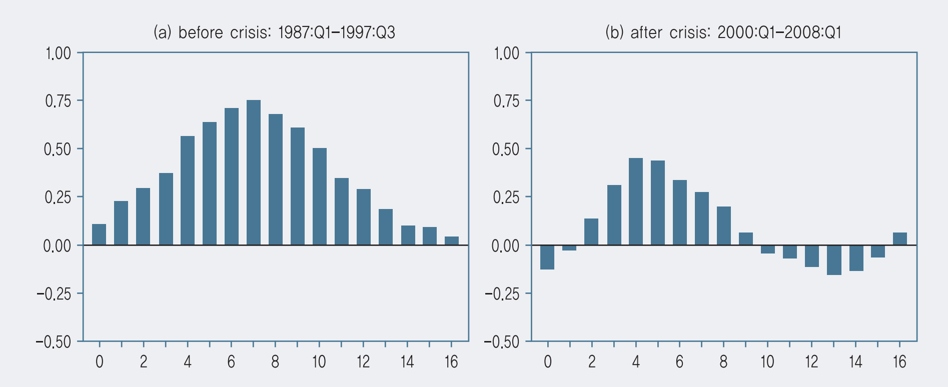
$$\widetilde{P}_t^* = M_t \cdot V_t^* / Y_t^* \tag{1}$$

where $\widetilde{P}_t^*$ is P-star, V_t^* the long-run equilibrium velocity, and Y_t^* long-run equilibrium real GDP. The long-run equilibrium velocity and long-run equilibrium real GDP can be calculated using various methods. I use HP-filter in this study to estimate these two.

Comparing P-star estimated using M2 with the actual price level, P-star seems to have been very useful during the period before the Asian financial crisis. As seen in <Figure1>, at a time when P-star was greater than the actual price level (price gap>0), inflation accelerated.[2] In contrast, inflation has been mild when P-

2) The price gap is estimated with $\log(\widetilde{P}_t^* / P_t)$

Figure 2 Cross correlation between price gap and inflation[1]



Note: 1) Correlation coefficient between the price gap in period t and GDP deflator inflation(year-over-year) in period t+j

star was lower than the actual price level (price gap <0). However, since the financial crisis, this relationship has weakened with the exception of a period after mid 2003. Even the correlation between the price gap and the rate of inflation shows that the relationship between the two variables has significantly weakened. According to the <Figure 2>, the price gap before the crisis had a high prediction power for inflation. The expansion of the price gap is highly correlated with inflation after four to nine quarters periods. However, the correlation between these two has become significantly lower since the financial crisis.[3]

This change is not a phenomenon limited only to Korea but one commonly observed in other countries whose monetary policy regimes shifted from monetary targeting to inflation targeting: New Zealand, Australia, Canada, Spain, and the U.K being cases in point.

Cross correlations between the price gap and inflation for each country are displayed in <Figure 3> and <Figure 4>. In calculating the price gap in these countries, I apply the same method for the estimation of the long-run equilibrium velocity and long-run equilibrium real GDP as in Korea. The analytical results

3) The period after the financial crisis is the one since 2000. This is because the relationship between price gap and inflation are relatively more stable during the period since 2000 rather than from 1999. The massive fund movement between financial products right after the crisis could disrupt a price gap that is estimated with M2. That is, the Lf (the liquidity of financial institution) increase has been slowed down the after the crisis, but M2 has expanded, having led to the decrease in the velocity that was estimated with M2 during 1998 to 1999. For more detail, refer to Kim (2008, pp, 59-61).

for four of the countries other than New Zealand are presented in <Figure 3>. For Australia, the price gap's predictive power for inflation was found to be not very large during the period when a monetary targeting regime was in place. However, the predictability has more deteriorated during the period of the policy transition into inflation targeting. The time lag structure between the price gap and future inflation in Canada, Spain and the U.K. differs depending on countries. However, during the period when monetary targeting was adopted, the correlation between two variables is generally very high. In contrast, the implementation of inflation targeting, the correlation between these two diminished sharply in these countries.

New Zealand carried out a monetary targeting policy from 1985 to 1989, but cross correlation between two variables are not significant during the corresponding period due to limitations in sample size.[4)] For that reason, <Figure 4> provides cross correlation between the price gap and inflation only for the period when inflation targeting was carried out. The figure shows the price gap and inflation has relationship with inflation after 2 years, but the degree is extremely low. In this regard, it is unclear how the relationship between the two variables has changed, but even for New Zealand it is found that the price gap has an extremely low power for predicting inflation under inflation targeting.

In this study, I argue this change stemmed mostly from the shift in monetary policy. To provide this assertion with theoretical support, it is essential to examine what factors determine prices in the long-run. Thus, in the next section, I discuss the theory of price level determination which analyzes the determinants of prices in the long-run.

4) The GDP data of New Zealnd in the statistics released by IMF and OECD are backdated only to the second quarter of 1987.

Figure 3 **Cross-country comparison: cross correlation between price gap and inflation**[1)]

Notes: 1) Correlation coefficient between the price gap in period t and GDP deflator inflation (year-over-year) in period t+j

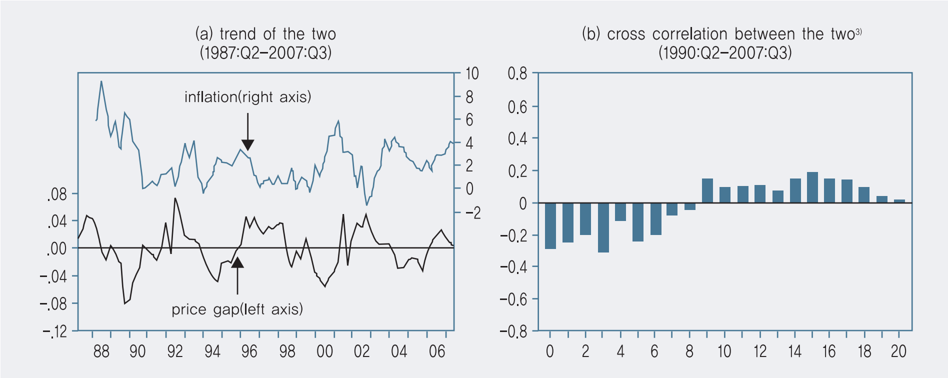
2) Correlation coefficients for some periods are not computed due to limitation on the sample size.

3) The monetary aggregate and the sample period for each country are as follows:

Australia: M3,	1976:Q1-1992:Q4(monetary targeting period),
	1993:Q1-2007:Q4(inflation targeting period)
Canada: M2++ Gross,	1975:Q1-1981:Q4(monetary targeting period),
	1991:Q1-2007:Q4(inflation targeting period)
Spain: M3,	1972:Q1-1994:Q1(monetary targeting period),
	1994:Q2-1998:Q4(inflation targeting period)
U.K.: M4,	1982:Q2-1989:Q4(monetary targeting period),
	1992:Q4-2007:Q3(inflation targeting period)

Sources: International Financial Statistics, IMF

Figure 4 **Price gap[1] and inflation[2] in New Zealand**



Notes: 1) M3 Broad is used to measure the price gap.
2) GDP deflator inflation (year-over-year)
3) Cross correlation between the price gap in period t and inflation (year-over-year) in period t+j
Source: International Financial Statistics, IMF

III. Determination of the Long-run Equilibrium Price Level

In this section, I look into what factors determine prices in the long-run. To this end, I set up a dynamic DSGE (Dynamic Stochastic General Equilibrium) model consisting of households, firms, the government and the central bank. In order to analyze what factors determine prices in the long-run, it is necessary to find out the long-run equilibrium, defined as the equilibrium that emerges on condition that all prices (including prices of production factors) are flexible. In addition, the policy regimes that should be considered include monetary targeting and the interest rate rule in order to find out how the determinants of prices change in accordance with the policy regimes implemented by central banks. When introducing money in economic models, Leeper (1991) assumes money into the utility function while Sims (1994) models the transaction costs and Woodford (1994) uses the cash-in-advance constraint.

In this paper, I use the assumption of money in the utility function as Leeper does. However, I explain the determinants of prices in accordance with policy regimes in a consistent way. In addition, I generalize price dynamics by considering an economy with technology growth.

1. Model Economy

Economy is composed of households, firms, the government and the central bank. In the household sector, there is a representative household that consumes product (C_t) and supplies its labor (H_t). Firms supply a continuum of differentiated products ($Y_t(i)$) and set prices ($P_t(i)$). To simplify the discussion, I assume that there is only one factor of production, that is, labor ($L_t(i)$). The government provides transfers (TR_t) to the household sector and finances its expenditures by taxing (T_t) and issuing bonds (B_t). However, I assume that the government consumes no products. The central bank conducts its monetary policy by controlling money supply (M_t) or the interest rate (R_t) in line with its policy regime.

The household sector is assumed to maximize its life-time utility as follows:

$$Max.\ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t + \psi_t^m \log\left(\frac{M_t}{P_t}\right) - \frac{\psi_h}{1+\gamma} H_t^{1+\gamma} \right\} \quad (2)$$

$$s.t.\ \ P_t C_t + M_t + B_t = W_t H_t + D_t + P_t \tau_t + M_{t-1} + R_{t-1} B_{t-1}$$

where β is the discount factor, ψ_t^m a time-varying parameter which affects the preference for money, ψ_h and γ parameters that are related to the preference for labor.

$C_t = \left[\int_0^1 C_t(i)^{(\epsilon-1)/\epsilon} di\right]^{\epsilon/(\epsilon-1)}$ is a CES aggregator, P_t the price level, W_t the nominal wage rate, τ_t net transfer ($= TR_t - T_t$), and $D_t = \int_0^1 D_t(i) di$, the profits that the household receives from firms in which $D_t(i)$ is the profits of ith firms.).

An individual firm chooses an optimal price in order to maximize its profits subject to the production function and its demand schedule.

$$Max.\ D_t(i) = P_t(i) Y_t(i) - W_t L_t(i) \quad (3)$$

$$s.t.\ \ Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t,\ Y_t(i) = A_t L_t(i)$$

where Y_t the aggregate output of the economy and A_t is the technological level)

The government finances its expenditures by taxing and issuing government bonds and uses the fund for transfers and payment of the principal and interest on government bonds. In this process, it faces the following budget constraint:

$$P_tT_t + B_t^T = P_tTR_t + R_{t-1}B_{t-1}^T \tag{4}$$

where, B_t^T is the total amount of government bonds issued which include all the bonds held by the household sector and the central bank.

The central bank adjusts monetary supply and the interest rate by selling bonds in an open market and is faced with the following budget constraint.

$$B_t^c - R_{t-1}B_{t-1}^c = M_t - M_{t-1} \tag{5}$$

where B_t^c is government bonds held by the central bank.

The budget constraint of the combined government sector, which includes the government and the central bank, is expressed as follows:

$$B_t + M_t - M_{t-1} = P_t\tau_t + R_{t-1}B_{t-1} \tag{6}$$

where $B_t = B_t^T - B_t^c$ is government bonds that would be held by households.

The first order conditions for maximizing the utility of households and the transversality conditions are as follows.

$$\frac{1}{C_tP_t} = \beta R_t E_t \frac{1}{C_{t+1}P_{t+1}} \tag{7}$$

$$\psi_h C_t H_t^\gamma = \frac{W_t}{P_t} \tag{8}$$

$$\psi_t^m \frac{1}{M_t/P_t} = \left(1 - \frac{1}{R_t}\right)\frac{1}{C_t} \tag{9}$$

$$\lim_{T\to\infty} E_t\beta^T \frac{1}{C_TP_T} B_T = 0 \tag{10}$$

$$\lim_{T\to\infty} E_t\beta^T \frac{1}{C_TP_T} M_{T-1} = 0 \tag{11}$$

The first order condition for an individual firms to maximize its profits is:

$$P_t(i) = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{A_t} = \mu^p \frac{W_t}{A_t} \tag{12}$$

where, $\mu^p = \dfrac{\epsilon}{\epsilon - 1}$ is the desired mark-up ratio. In equation (12), the desired mark-up ratio and marginal costs are identical across all firms. Therefore, all prices are identical when prices are flexible. As a consequence, under the symmetric equilibrium $P_t(i) = P_t,\ Y_t(i) = Y_t\ \ \forall(i)$, in which the prices and output of all firms are equal, the following equilibrium equation is derived:

$$\frac{W_t}{P_t} = \frac{A_t}{\mu^p},\ Y_t = A_t L_t \tag{13}$$

Moreover, since aggregate demand (C_t) and aggregate supply (Y_t) should be identical with each other in the equilibrium, and the demand (L_t) for and supply (H_t) of labor should match, we have:

$$H_t = L_t = \frac{Y_t}{A_t} = \frac{C_t}{A_t},\ \psi_h C_t \left(\frac{C_t}{A_t}\right)^{\gamma} = \frac{W_t}{P_t} = \frac{A_t}{\mu^p} \tag{14}$$

Consequently, equilibrium production and labor are:

$$Y_t^* = C_t^* = (\psi_h \mu^p)^{-\frac{1}{1+\gamma}} A_t,\ H_t^* = L_t^* = (\psi_h \mu^p)^{-\frac{1}{1+\gamma}} \tag{15}$$

The production level given in equation (15) is called the natural rate of output or potential output that is proportional to the technology level. Labor takes on a certain value that is affected by parameters related to labor preferences, and the mark-up.

In the analysis above, none of the real variables ($C_t,\ Y_t,\ H_t,\ L_t,\ W_t/P_t$) with the exceptions of real money balance (M_t/P_t) and the real value of bond issuance (B_t/P_t), are affected by money stock or interest rate but are determined only by supply factors such as the technological level. In verifying the existence of the equilibrium of the overall price level (P_t) and other variables whose equilibrium values are not determined, and whether they are unique, equations (7) and (9) and the transversality conditions (10) and (11) play important roles. Meanwhile, as income generated from production is fully distributed and aggregate output becomes equal to aggregate consumption, this implies that the budget constraint equation of the household sector is identical to that of the combined government sector. As a result, the following five equilibrium conditions act as essential factors in explaining the equilibrium of the price level.

$$\frac{1}{C_t P_t} = \beta R_t E_t \frac{1}{C_{t+1} P_{t+1}} \tag{7}$$

$$\psi_t^m \frac{1}{M_t / P_t} = \left(1 - \frac{1}{R_t}\right) \frac{1}{C_t} \tag{9}$$

$$B_t + M_t - M_{t-1} = P_t \tau_t + R_{t-1} B_{t-1} \tag{6}$$

$$\lim_{T \to \infty} E_t \beta^T \frac{1}{C_T P_T} B_T = 0 \tag{10}$$

$$\lim_{T \to \infty} E_t \beta^T \frac{1}{C_T P_T} M_{T-1} = 0 \tag{11}$$

Meanwhile, this study considers the case where the economy grows in line with the technological advances. When there are technological advances, consumption, production, real wages, real money balances, and the real value of bond issuance become the non-stationary variables that are commonly affected by the level of technology. In this case, stationary variables are defined as follows:

$$m_t \equiv \frac{M_t}{P_t C_t}, \ b_t \equiv \frac{B_t}{P_t C_t}, \ \tilde{\tau}_t \equiv \frac{\tau_t}{C_t}$$

Accordingly, the five equilibrium conditions above are expressed as follows:

$$m_t = \beta R_t E_t \left[m_{t+1} \frac{M_t}{M_{t-1}} \right] \tag{16}$$

$$m_t = \psi_t^m \frac{R_t}{R_t - 1} \tag{17}$$

$$b_t + m_t - \frac{m_{t-1}}{\Pi_t \Gamma_t} = \tilde{\tau}_t + R_{t-1} \frac{b_{t-1}}{\Pi_t \Gamma_t} \tag{18}$$

$$\lim_{T\to\infty} E_t \beta^T b_T = 0 \tag{19}$$

$$\lim_{T\to\infty} E_t \beta^T m_T \frac{M_{T-1}}{M_T} = 0 \tag{20}$$

where $\Pi_t \equiv P_t / B_{t-1}$ and $\Gamma_t \equiv A_t / A_{t-1} = C_t / C_{t-1}$.

Under the general setup of the model economy, the determinants of the price level differ in accordance with the policy regime. This will be discussed in the next section.

2. Theory of Price Level Determination by Policy Regime

In discussing price level determination, I first deal with the perfect foresight model which has no stochastic elements. After that I explain the stochastic case. In the perfect foresight model, the rate of technological advances (Γ_t), the preference for money (ψ_t^m) and the monetary policy instrument (μ_t or R_t) have constant values.

$$\Gamma_t \equiv \frac{A_t}{A_{t-1}} = \Gamma,\ \psi_t^m = \psi^m,\ \mu_t \equiv \frac{M_t}{M_t - 1} = \mu \text{ (or } R_t = R\text{)}$$

In contrast, in the stochastic environment, the exogenous stochastic variables in the economy are assumed to have the following process.

$$\log\Gamma_t = \log\frac{A_t}{A_{t-1}} = \log\Gamma + \zeta_t^a \qquad \text{(technology growth)} \tag{21}$$

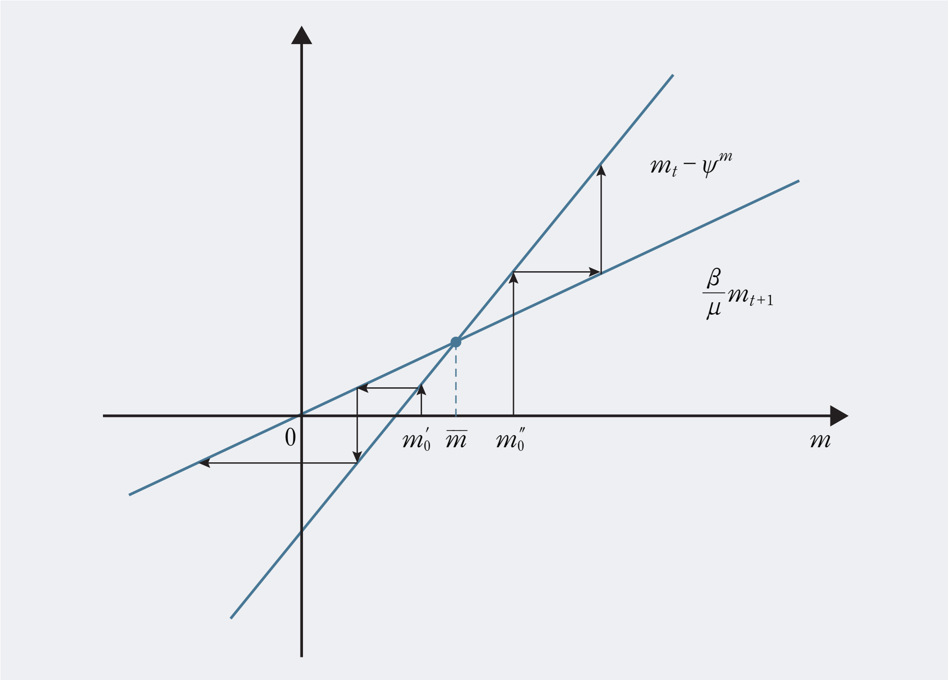
$$\log\frac{\psi_t^m}{\psi^m} = \rho\log\frac{\psi_{t-1}^m}{\psi^m} + \zeta_t^{md} \qquad \text{(preference for money)} \tag{22}$$

$$\log\frac{\mu_t}{\mu} = \phi_\mu \log\frac{\mu_{t-1}}{\mu} + \zeta_t^{ms} \qquad \text{(money growth)} \tag{23}$$

$$R_t = R + \alpha(\Pi_t + \Pi^*) + \zeta_t^r,\ \zeta_t^r = \phi_r \zeta \zeta_{t-1}^r + \zeta_t \qquad \text{(interest rate rule)} \tag{24}$$

where ζ_t^a, ζ_t^{md}, ζ_t^{ms} and ζ_t are mean-zero stochastic processes with their standard deviations α_a, α_{md}, α_{ms} and α_r, respectively.

Figure 5 **Equilibrium of money-income ratio(monetary targeting)**



A. Monetary Targeting

(1) Perfect Foresight

Under this policy regime, the central bank adopts a policy that maintains a constant growth rate of money as follows:

$$\frac{M_t}{M_{t-1}} = \mu \quad \text{or} \quad M_t = M_0 \mu^t$$

where only the case $\mu > \beta$ is considered.5)

Since $R_t = m_t / (m_t - \psi^m)$ is derived from the equation (17), equation (16) becomes the following difference equation whose equilibrium can be explained by <Figure 5>.

5) The reason to employ the assumption of $\mu > \beta$ is to exclude the case equilibrium doesn't exist

$$m_t - \psi^m = \frac{\beta}{\mu} m_{t+1} \tag{25}$$

In this figure, the equilibrium of the money to income ratio (m_t) is grouped into a steady state equilibrium and a non-steady state equilibrium. When the money to income ratio is $\overline{m}$, it is a steady state equilibrium, which satisfies the transversality condition (20). When the current money to income ratio is smaller than $\overline{m}$ (for example, m_0'), the ratio may decrease continuously to a negative value, which cannot be an equilibrium.[6)] When the current money to income ratio is greater than $\overline{m}$ (for example, m_0''), money to income ratio steadily increases. When $\mu \geq 1$, transversality condition (2) is not satisfied. Thus this cannot be an equilibrium.

On the other hand, when $\beta < \mu < 1$, the transversality condition (20) is satisfied and this process can be an equilibrium. In addition, there are numerous cases where the current money to income ratio is greater than $\overline{m}$, and for each case, the process of steady increase in the money to income ratio can be an equilibrium. As a consequence, there could exist a large number of non-steady state equilibra. These are cases where the price level decreases at a more rapid pace even if money supply decreases ($\beta < \mu < 1$), leading to an increase in the money to income ratio This is the hyper-deflation pointed out by Sims (1994). Freidman (1969) argues that a deflationary policy that reduces money supply in order to maintain zero nominal interest rates could maximize social welfare. However, Woodford (1994) claims that there exist a number of money to income ratios for this case, generating the problem of the indeterminacy of the price level.

In order to tackle this multiple equilibria, it is necessary to keep money supply at a certain level or increase it. In this study, therefore, I consider only the case where $\mu \geq 1$, in which $\overline{m}$ is the unique equilibrium of money to income ratio. $\overline{m} = \dfrac{\psi^m \mu}{\mu - \beta}$ can be derived from equation (25), and as a consequence, the equation for the equilibrium price level is derived as follows:

$$P_t = \frac{\mu - \beta}{\psi^m \mu} \frac{M_t}{HA_t} \tag{26}$$

where $H \equiv (\psi_h \mu^p)^{-\frac{1}{1+\gamma}}$. This theoretically supports the conventional Quantity

6) This is the infeasibility condition mentioned by Obstfeld and Rogoff (1983).

Theory of Money. When aggregate supply in an economy increases thanks to technological advance, prices fall while prices rise in response to an increase in money supply. And according to the equation above, the inflation rate becomes the difference between the growth rate of money and the rate of technological advance $\Pi_t = \mu/\Gamma$. The results of the discussion above, reveal not just the importance of money supply in the determination of price level as well but also suggest a inflation is a monetary phenomenon in that excessive money supply causes inflation.

The equilibrium in the entire economy is finally determined when the equilibrium condition in the fiscal aspect is satisfied. To confirm this, it is necessary to analyze equation (18) and the transversality condition (19). After substituting the variables determined in the monetary aspect, the equation (18) is expressed as a 1st order difference equation as follows.[7]

$$b_t = \frac{1}{\beta} b_{t-1} + \tilde{\tau}_t - \overline{m}\left(1 - \frac{1}{\mu}\right) \tag{27}$$

If the government conducts its fiscal policy in such a way as to maintain a constant ratio of net transfer to GDP ($\tilde{\tau}_t = \frac{\tau_t}{C_t} = \frac{\tau_t}{Y_t} = \overline{\tau}$), b_t expand at a pace of $1/\beta$, failing to satisfy transversality condition (19). This implies households no longer purchase government bonds, as a result of which this situation cannot be an equilibrium, and the determination of the price level cannot be guaranteed. However, if the government conduct its fiscal policy by adjusting the net transfer to GDP ratio in response to an increase in the size of the government debt ($\tilde{\tau}_t = \overline{\tau} - \phi b_{t-1}$), the equation (23) is derived as follows.

$$b_t = \left(\frac{1}{\beta} - \phi\right) b_{t-1} + \overline{\tau} - \overline{m}\left(1 - \frac{1}{\mu}\right)$$

In this equation, if ϕ satisfies a certain condition for example, ($1 < 1/\beta - \phi < 1$) the transversality condition (19) holds.

According to Woodford (1995, p.27), a fiscal policy that guarantees the transversality condition (19) to be valid regardless of the behavior of other endogenous variables within an economy is a Ricardian fiscal policy regime

7) It should be noted that $1 = \beta R/\Pi\Gamma$ GAMMA under steady state in the Euler equation in order to derive the equation (27).

whereas a fiscal policy that does not meet such a condition is categorized as non-Ricardian fiscal policy regime. Meanwhile, satisfying the transversality condition (19) implies that it is possible to repay all the current government debt from the current and future budget surpluses, This, in turn means that fiscal soundness will be maintained from a long-term perspective. Based on this classification, the fiscal policy ($\tilde{\tau}_t = \bar{\tau}$) discussed above is a kind of non-Ricardian fiscal policy regime, while the latter ($\tilde{\tau}_t = \bar{\tau} - \phi b_{t-1}$) is a Ricardian fiscal policy regime. Therefore, the analytical results above imply that even if fiscal variables do not affect the price level, it is necessary to adopt a Ricardian fiscal policy regime that maintains fiscal soundness from a long-term perspective in order for the price level to be determined from the monetary aspect.

(2) Stochastic Environment

Under a stochastic environment, it is not easy to describe the equilibirum solution with a global analysis used so far. For this case, it is possible to look into the equilibrium by a local analysis that applies log-linearization near the steady state.

When using log-linearization, the exogenous stochastic processes in the economy are:

$$\hat{\Gamma}_t = \zeta_t^a \quad \text{(technology growth)} \tag{21'}$$

$$\widehat{\psi_t^m} = \rho \widehat{\psi_{t-1}^m} + \zeta_t^{md} \quad \text{(preference for money)} \tag{22'}$$

$$\hat{\mu}_t = \phi_\mu \widehat{\mu_{t-1}} + \zeta_t^{ms} \quad \text{(money growth)} \tag{23'}$$

where the hat above variables represents a percentage deviation of the variables from the steady state values.

Under exogenous money supply, log-linearized forms for equation (16) and (17) and the ratio of money to income are:

$$\widehat{m_t} = \hat{R}_t + E_t \widehat{m_{t+1}} - \phi_\mu \hat{\mu}_t \tag{28}$$

$$\widehat{m_t} = \widehat{\psi_t^m} - \frac{1}{R-1} \hat{R}_t \tag{29}$$

$$\widehat{\Pi_t} = \hat{\mu}_t - \hat{\Gamma}_t - (\widehat{m_t} - \widehat{m_{t-1}}) \tag{30}$$

Finally, the equilibrium solution for the money to income ratio obtained by using the equation (28) and (29), and the equilibrium solution for inflation is derived by substituting this ratio into equation (30):

$$\widehat{m}_t = \frac{R-1}{R-\rho}\widehat{\psi}_t^m - \frac{\phi_\mu}{R-\phi_\mu}\widehat{\mu}_t \tag{31}$$

$$\widehat{\Pi}_t = \frac{R}{R-\phi_\mu}\widehat{\mu}_t - \frac{\phi_\mu}{R-\phi_\mu}\widehat{\mu_{t-1}} - \widehat{\Gamma}_t - \frac{R-1}{R-\rho}(\widehat{\psi_t^m} - \widehat{\psi_{t-1}^m}) \tag{32}$$

Equation (32) means that, when money supply exceeds the additional demand for money resulting from technological advances and shifts in the preference for money, inflation emerges. However, unlike perfect foresight, current inflation is affected not just by the current growth rate of money supply, but also by the money growth rate in the previous period. According to equation (29), the demand for money (money to income ratio, that is, the demand for money per one unit of income) is influenced by the nominal interest rate as well as the preference for money. In general, the nominal interest rate is affected by the inflation rate in the next period. If money growth is persistent, current money supply ($\widehat{\mu}_t$) affects money supply in the next period ($\phi_\mu\widehat{\mu}_t$), influencing the inflation in the next period. As a result, this increases the current nominal interest rate and it acts as a factor to decrease the current demand for money(equation (31)). By the same logic, the growth rate of money in the previous period leads to a decrease in the demand for money in the previous period, Thus, the growth rate of money affects the demand for money in the previous period ($\widehat{m_{t-1}}$,) in equation (30), which in turn influences the current inflation. Therefore, the equation (32) implies that, if defining the long-run equilibrium price as the price level that appears when all prices within an economy are flexible variations in money supply determines the long-run equilibrium price under monetary targeting.

B. Interest Rate Rule

(1) Perfect Foresight

The central bank is assumed to set a nominal interest rate as follows:

$$R_t = \max[1,\ R + \alpha(\Pi_t - \Pi^*)] \tag{33}$$

$$= \max\left[1,\ R + \alpha(\Pi_t - \frac{\beta R}{\Gamma})\right]$$

where it is assumed that $\alpha\beta/\Gamma > 1$. The equation above implies that the lower board of the nominal interest rate is set to one in order to prevent the situation when the nominal interest rate (R_t) is smaller than one due to the existence of feedback term. In addition, as $\Pi = \beta R/\Gamma$ at the steady state, this means that, in circumstances technological advance and discount factor are exogenously given, if either one of the targeting inflation rate (Π^*) and steady state nominal interest rate (R) is determined, the other is given by this equation.

Under this interest rate rule of the central bank, the equation (17) still holds, and as a consequence, $R_t = m_t/(m_t - \psi^m)$ is valid. By substituting into equations (16) and (33), the following two equation is derived

$$m_t - \psi^m = \beta m_{t+1}\left(\frac{M_t}{M_{t+1}}\right) \tag{34}$$

$$\frac{m_t}{m_t - \psi^m} = \max\left[1,\ (R - \alpha\Pi^*) + \frac{\alpha}{\Gamma}\frac{M_t}{M_{t-1}}\frac{m_{t-1}}{m_t}\right] \tag{35}$$

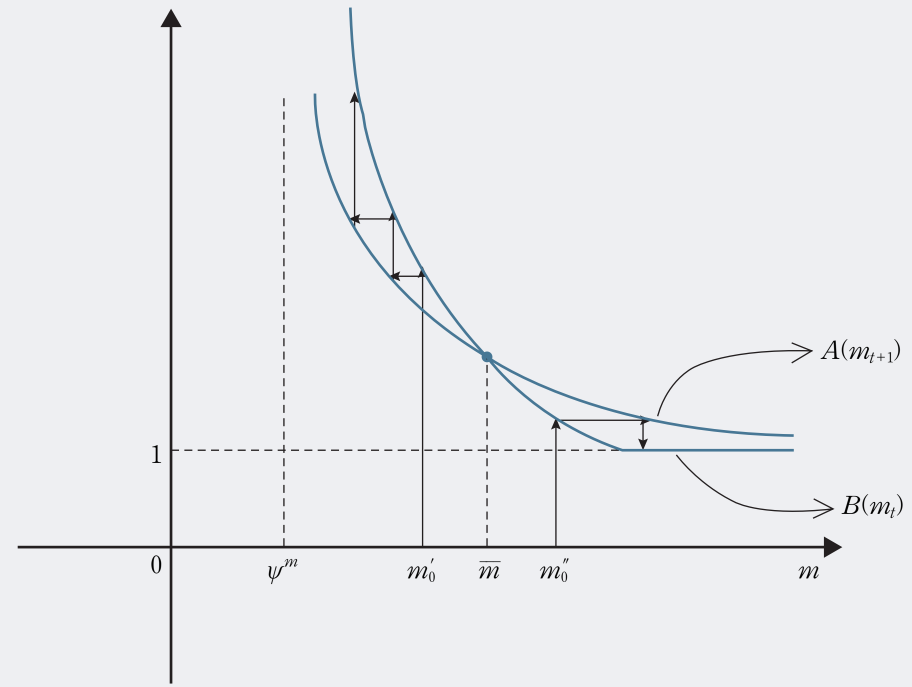
From equation (34), $M_{t+1}/M_t = \beta m_{t+1}/(m_t - \psi^m)$ and by shifting the timing of the equation to one period forward (t+1):

$$\frac{m_{t+1}}{m_{t+1} - \psi^m} = \max\left[1,\ (R - \alpha\Pi^*) + \frac{\alpha\beta}{\Gamma}\frac{m_t}{m_t - \psi^m}\right] \tag{36}$$

Defining the left and right sides of the equation (36) as $A(m_{t+1})$ and $B(m_t)$, respectively, it is possible to see how the equilibrium money to income ratio is determined in <Figure 6>. It is assumed $\alpha\beta/\Gamma > 1$. Unless this condition is satisfied, there can be a large numbers of equilibriums. This condition is a generalization of the condition $\alpha\beta > 1$ that is provided assuming there is no technological advance.

Even when $\alpha\beta/\Gamma > 1$, the equilibrium of money to income ratio could possibly be a both the steady-state equilibrium and a non-steady state equilibrium. When the money to income ratio is $\overline{m}$, this is a steady state equilibrium, which satisfies the transversality condition (20). When the money

Figure 6 Equilibrium of money-income ratio(interest rate rule)



to income ratio is smaller than $\overline{m}$ (for example, m_0'), the money to income ratio converges to ψ^m. This process satisfies the transversality condition (20), and this can be an equilibrium (non-steady state equilibrium). When the current money to income ratio is greater than $\overline{m}$ (for instance, m_0''), the nominal interest rate (R_t) eventually becomes 1, leading to the situation that there exists no money to income ratio to satisfy this. Therefore, this process cannot be an equilibrium. A non-steady state equilibrium in which money to income ratio continuously decreases, implies the hyperinflation where inflation rate keeps rising relatively rapidly compared to the growth rate of money. Obstfeld and Rogoff (1983) argue that, for the prevention of hyperinflation, it is necessary to implement the fractional backing system in which, the government exchanges asset having a real value with money, when the price level reaches the preset level. This paper also assumes that a government is implicitly or explicitly implementing the system advocated by Obstfeld and Rogoff (1983) and focuses only on the steady state equilibrium.

Under this condition, the money to income ratio is uniquely determined at

$\overline{m} = \psi^m R/(R-1)$. Therefore, equation (35) reduces to the following, which implies that the current money stock is determined by this equation because the money stock in the previous period, is a predetermined variable.

$$\frac{\overline{m}}{\overline{m}-\psi^m} = (R-\alpha\Pi^*) + \frac{\alpha}{\Gamma}\frac{M_t}{M_{t-1}} \Rightarrow M_t = \Pi^*\Gamma M_{t-1} = \beta R M_{t-1}$$

In addition, as the current money supply is determined and the money to income ratio is constant, the current price level is determined as:

$$P_t = \frac{\beta(R-1)}{\psi^m}\frac{M_{t-1}}{HA_t} \tag{37}$$

Thus, If the central bank uses an interest rate rule, the current price level can be determined by the policy variables of the central bank (Π or R). This implies that the monetary theory of the price level is applied under this policy regime. However, even in this case, Ricardian fiscal policy should be undertaken in order to support the equilibrium of the entire economy. In this regard, policy coordination by the government is called for including the implementation of a fractional backing system and the maintenance of fiscal soundness

(2) Stochastic Environment

Under a stochastic environment, it is necessary to examine the equilibrium solution through log linearization near the steady state. In this case, the equation (21′) and (22′) should be directly used as the exogenous stochastic processes for technology growth and the preference for money. However, the following equation obtained from log-linearization of the equation (24) should be used as the stochastic process for the monetary policy,

$$\hat{R}_t = \frac{\alpha\beta}{\Gamma}\hat{\Pi}_t + \zeta_t^r,\ \zeta_t^r = \phi\zeta_{t-1}^r + \xi_t \quad \text{(interest rate rule)} \tag{24'}$$

where $\alpha\beta/\Gamma > 1$.

Under the interest rate rule, money is supplied endogenously to accomodate the demand for money. Thus, the equilibrium condition is summarized with the interest rate rule and the following three equations.

$$\widehat{m}_t = \widehat{R}_t + E_t\widehat{m_{t+1}} - E_t\widehat{\mu_{t+1}} \tag{38}$$

$$\widehat{m}_t = \widehat{\psi}_t^m - \frac{1}{R-1}\widehat{R}_t \tag{39}$$

$$\widehat{\mu}_t = \widehat{\Pi}_t + \widehat{\Gamma}_t + (\widehat{m}_t - \widehat{m_{t-1}}) \tag{40}$$

By shifting the timing of equation (40) one period forward $(t+1)$ and is substituted into the equation (38), the nominal interest rate becomes equal to the expected inflation ($\widehat{R}_t = E_t\widehat{\Pi_{t+1}}$). By applying this relationship to the interest rate rule, the following first order difference equation for inflation is derived:

$$\widehat{\Pi}_t = \frac{\Gamma}{\alpha\beta}E_t\widehat{\Pi_{t+1}} - \frac{\Gamma}{\alpha\beta}\zeta_t^r \tag{41}$$

So it turns out that current inflation has a negative relationship with the interest rate shock.[8)]

$$\widehat{\Pi}_t = -\frac{\Gamma}{\alpha\beta - \Gamma\phi_r}\zeta_t^r \tag{42}$$

The bottom line of the analysis so far is that the long-run equilibrium inflation rate, and thus, the long-run equilibrium price level, are determined by variations in the interest rate. Thus, the monetary theory of price level determination is applied to this regime.

IV. Price Gap's Predictive Power for Inflation

The analysis in the previous section shows how the equilibrium price level is determined when all prices within an economy are flexible. Such an equilibrium price level only appears in the long-run, which is not observed in the actual economy. In the actual economy, the existence of price rigidities could generate

8) The equilibrium solution for the nominal interest is $\widehat{R}_t = -[\tau\phi_r/(\alpha\beta - \tau\phi_r)]\zeta_t^r$. This is used in explaining the movement of the nominal interest rate in the next section's the analysis of impulse response functions.

a discrepancy between the actual price level and the long-run equilibrium price level. In this section, I analyze how the actual price level moves differently from the long-run equilibrium and whether the actual price level converges to the long-run equilibrium price level with the passage of time.

1. Sticky Price Model

To explain the price dynamics when there are nominal rigidities in prices and wages, it is essential to modify the optimization behaviors by households and firms. First, I assume that households provide differentiated labors and set the wages for their labors with a certain probability $(1-\theta_w)$.[9] Next, I follow Calvo (1983) to describe the price-setting behavior of individual firms. Each firm adjusts the price of its product with a certain probability $(1-\theta)$ in this model. In addition, I introduce rule-of-the-thumb firms' (portion of them within an economy being) as in Gali and Gertler (1999) in order to explain inflation inertia.

The following log-linearized equations are required in order to explain the movements of the whole economy.[10]

$$\widehat{c_t} = -\widehat{R_t} + E_t\widehat{c_{t+1}} + E_t\widehat{\Pi_{t+1}} + E_t\widehat{\Gamma_{t+1}} \tag{43}$$

$$\widehat{m_t} = \widehat{\psi_t^m} + \widehat{c_t} - \frac{1}{R-1}\widehat{R_t} \tag{44}$$

$$\widehat{\Pi_t} = \beta_1 E_t\widehat{\Pi_{t+1}} + \beta_2\widehat{\Pi_{t-1}} + k\widehat{w_t} \tag{45}$$

$$\widehat{\Pi_t^w} = \beta E_t\widehat{\Pi_{t+1}^w} + \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\gamma)}(1+\gamma)\widehat{c_t} - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\gamma)}\widehat{w_t} \tag{46}$$

9) For more detail about the sticky wage model, please refer to Erceg et al (2000).

10) As the sticky price model also consider the case where there is technology growth, stationary variables are defined as follows.

$$c_t \equiv C_t/A_t,\ m_t \equiv M_t/(P_tA_t),\ w_t \equiv W_t/(P_tA_t)$$

Moreover, it should be noted that, $m_t \equiv M_t/(P_tA_t)$ defined in this section is different from $m_t \equiv M_t/(P_tC_t)$ defined in the previous section.

where the slope coefficients of the Phillips curve are in the following relationship with the structural parameter:

$$\beta_1 = \beta\theta/[\theta + \omega(1-\theta(1-\beta))],\ \beta_2 = \omega/[\theta + \omega(1-\theta(1-\beta))],$$
$$k = (1-\omega(1-\theta)(1-\beta\theta)/[\theta + \omega(1-\theta(1-\beta))])$$

Equation (43) denotes the consumption Euler equation, and the equation (44) implies the demand for money. Equation (45) represents the New Keynesian Phillips Curve and equation (46) is the Phillips curve for wage inflation $\widehat{\Pi}_t^w$. The exogenous stochastic process within an economy are expressed as equation (21″)-(23′) as in the previous section. However, for the interest rate rule, I assume the following policy reaction function which additionally reflects the response to output gap.

$$\hat{R}_t = \phi_\pi \widehat{\Pi}_t + \phi_y \widehat{c}_t + \zeta_t^r,\ \zeta_t^r = \phi_r \zeta_{t-1}^r + \xi_t \qquad (47)$$

where $\phi_\pi = \dfrac{\alpha\beta}{\Gamma}$.

When all prices are flexible, the output gap is always zero ($\widehat{c}_t = 0$). So the policy reaction function above becomes identical to in the previous section.

2. Comparison between the Actual and Long-run Equilibrium Price Level

A. Parameterization

Under the existence of price rigidity, I look into how much between the actual price level differs from and the long-run equilibrium price level. To this end, the values of the structural parameters within the model are set as follows. I use standard values for β, γ, θ_w and ϵ_w (elasticity of substitution between differentiated labor). θ and ω are set to 0.75 and 0.4, respectively, with consideration of the parameter estimates of New keynesian Phillips Curve (for the period 1983.Q1～1996.Q4) by Kim and Ahn (2008). Meanwhile, Park and Moon (2005) show that the annul potential growth rate in Korea is estimated to be 6.9% during the period before the currency crisis (1991～1997) and 4.8%

during the period after the crisis (2001~2004). Moreover the annual growth rate of employment was 2.46% during the period before the crisis (1990~1996) and 1.62% during the period after the crisis (2001~2004). In view of these results, the annual growth rate of labor productivity is estimated to be 4.4%(6.9% − 2.46%) before the crisis while it is estimated to be 3.2% (4.8% − 1.62%) during the period after the crisis. Reflecting this, I set the rate of technological advance as follows. It is assumed that there is no technological advance ($\Gamma = 1$) in the impulse response analysis. In the simulation under monetary targeting, I set the rate of technological advance by converting to the annul growth rates of labor productivity before the crisis to the quarterly growth rate. In the simulation

Table 6 **Demand for risky assets (censored at 0.2)**

	parameter	value	description
parameters appearing in both regimes	β	0.99	discount factor
	γ	1	inverse of Frisch elasticity
	θ	0.75	probability of price stickiness
	ω	0.4	fraction of rule-of-thumb firms
	θ_w	0.64	probability of wage stickiness
	ϵ_w	21	elasticity of substitution among differentiated labor
	Γ	1,1.011,1.008	technology growth rate
	H	1	steady state hours worked
	R	$\Pi\Gamma/\beta$, $\Pi^*/\Gamma\beta$	steady state nominal interest rate
	ψ^m	$(R-1)/R$	preference for money demand
	ρ	0.9	persistence of preference for money demand
	σ_a	0.006	standard deviation of technology shocks
	σ_{md}	0.05	standard deviation of shocks to preference for money demand
parameters only for monetary targeting	μ	1, 1.05	steady state money growth rate
	Π	μ/Γ	steady state inflation rate
	ϕ_μ	0.4	persistence of money growth
	σ_{ms}	0.012	standard deviation of shocks to money growth
parameters only for interest rate rule	Π^*	1, 1.0075	target inflation rate
	$\phi_\pi = \alpha\beta/\Gamma$	1.5	coefficient on inflation gap in interest rate rule
	ϕ_y	0.5	coefficient on output gap in interest rate rule
	ϕ_r	0.8	persistence of shocks to interest rate
	σ_r	0.005	standard deviation of innovation term in interest rate shock

under the interest rate rule, I set the rate of technological progress by converting the annual growth rate of labor productivity after the crisis to the quarterly growth rate.

The other parameters common in the two policy regimes are set as follows. Hours worked at the steady state is normalized to be one. The parameter (ψ^m) that represents the preference for money is set so that the velocity of money becomes one at the steady state. The dstandard deviation of technology shocks is taken from Lee (2007). The persistence of the preference for money and the standard deviation of the money preference shock are set by the money demand function in Korea.

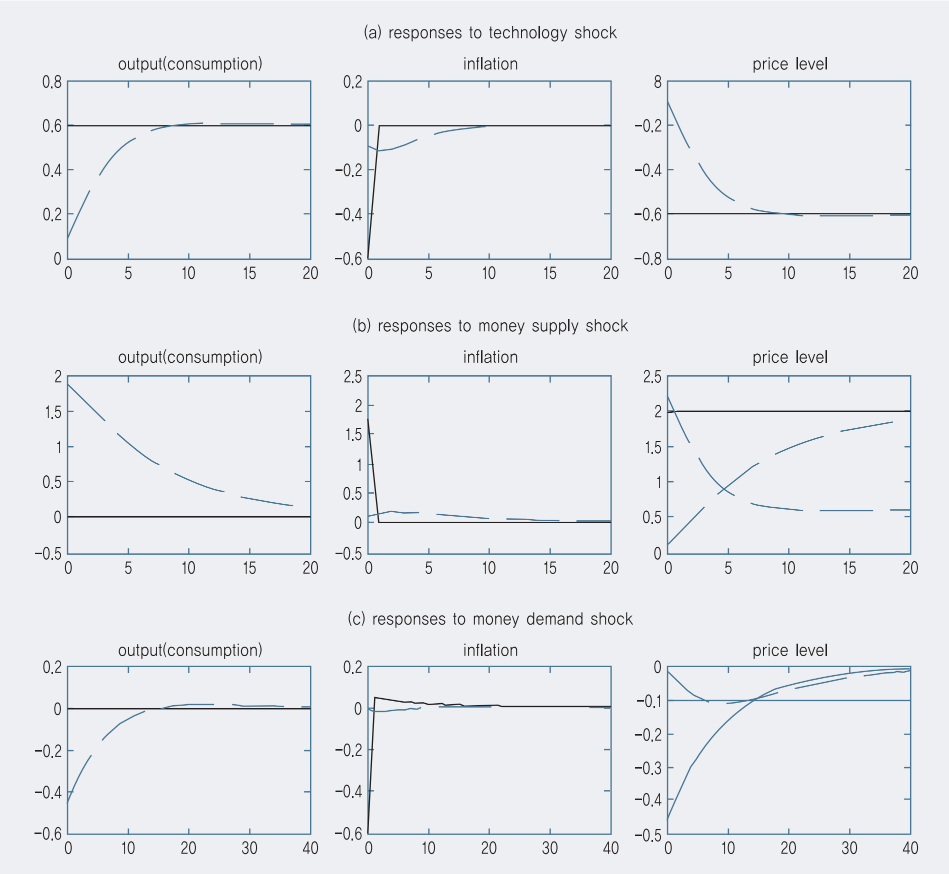
The parameters appearing only under monetary targeting are set as follows. For the steady growth rate of money, the persistence of money growth and the standard deviation of the money growth shock, I use the average growth rate, estimated coefficients and the standard error from the estimation of AR (1) model of the M2 growth rate for the period 1987:Q1～1997Q3 (but note that the steady state money growth rate is set to one in the impulse-response analysis). The values of parameters appearing under the interest rate rule are as follows. The target inflation rate is set at 3% on an annual basis and is converted to the quarterly rate (in the impulse-response analysis, the target inflation rate is set to one). For coefficients on the inflation and output gap of the interest rate rule, I use the standard values proposed by Taylor (1993). For the persistence and standard deviation of interest rate shocks, I employ estimates by Lee (2007). Other unexplained parameters are chosen to satisfy relationship among variables at the steady state.

B. Analysis of the Price Gap under Monetary Targeting

Let us examine the responses of economic variables to structural shocks. First, the impulse response functions under the monetary targeting are shown in <Figure 7>. If there is a technology shock, potential output immediately jumps up while actual output increases gradually, converging to potential output. Long-run equilibrium price immediately falls at the time a shock occurs, but if prices are sticky, the actual price level decreases slowly and the inflation rate is lowered over a long period of time. However, despite the different pace of the adjustment between the long-run equilibrium price and the actual price level, the actual price level ultimately converges to the long run equilibrium price.

When there is a shock to money growth, potential output remains the same, but actual output exceeds the potential output for a substantial period of time, thanks

Figure 7 **Impulse responses under monetary targeting**



Notes: 1) Horizontal axes show quarters after a shock. Vertical axes represent percent changes for output and the price level, and deviation from the steady state value for inflation.
2) Solid lines are responses when prices are flexible whereas dashed lines are responses when prices are sticky.

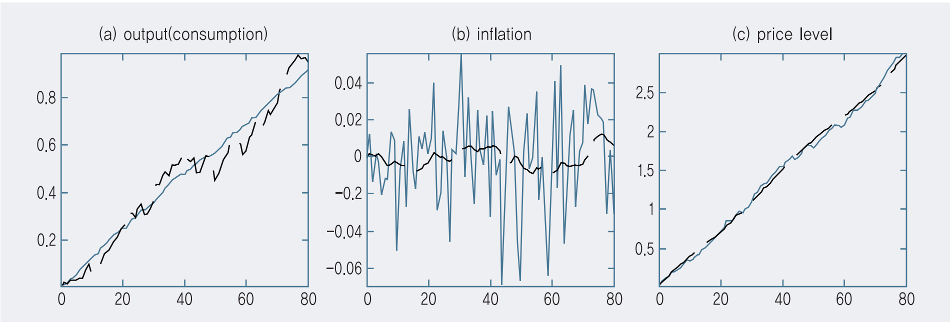
to the increase in aggregate demand. However, the effect subsidizes over time. As habit formation is not introduced in this model, monetary policy instantly affect output. Long-run equilibrium prices rise quickly, but if prices are sticky, inflation rises gradually and the actual price level converge to the long-run equilibrium price in the long-run.

Finally, when there is a shock to the preference for money, money supply become insufficient relative to money demand, leading to the occurrence of a tightening effect. Although potential output does not change, actual outputdecreases and then returns again to the pre-shock level. The long-run equilibrium prices immediately fall whereas the actual price level decreases slowly. However, as the preference for money goes back to the pre-shock level, both the long-run equilibrium price and the actual price level return its previous level.

The actual economic phenomenon is the process, in which, a number of shocks occur simultaneously at every moment and economic variables continuously respond to these shock. In order to compare the long-run equilibrium price with the actual price level under this circumstance, I conduct a stochastic simulation.[11)] According to the results exhibited in <Figure 8>, actual output moves up and downs centering on the potential output, and the inflation rate under flexible price fluctuate more sharply than the inflation rate under the sticky prices. Nevertheless, as is consistent with the results of the previous impulse response analysis the actual price level fluctuates closely the long-run equilibrium price.

In this regard, price gap between long-run equilibrium price (P_t^*) and the actual price level (P_t) contains very useful information for forecasting future inflation. When the long-run equilibrium price level is higher than the actual price level, inflation becomes higher and when the long-run equilibrium price is lower than actual price level, the inflation gets lower. Cross correlation coefficient between the price gap ($\log(P_t^*/P_t)$) and the inflation rate calculated from the simulation exhibits that the price gap's predictive power for inflation is very high.

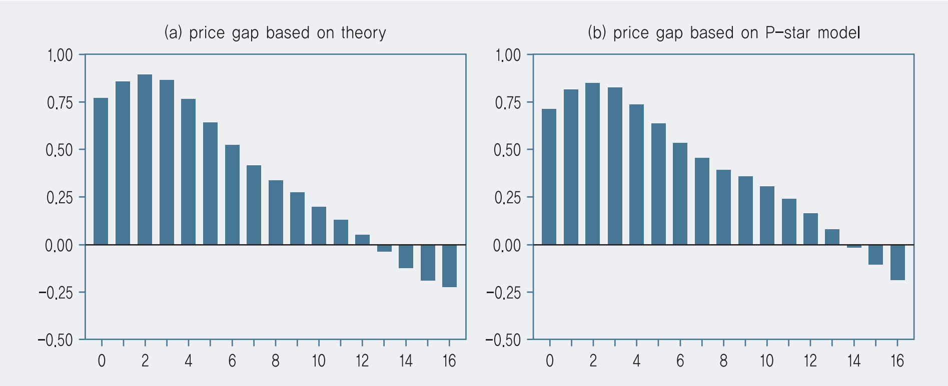
Figure 8 **Simulation result under monetary targeting**



Notes: 1) Horizontal axes show quarters after the initial period. Vertical axes represent log levels for output and the price level, and deviation from the steady state value for inflation.
2) Solid lines are responses when prices are flexible whereas dashed lines are responses when prices are sticky.

11) To this end, I generate shocks to technology, money growth and preference for many demand from independent normal distribution for 81 periods.

Figure 9 Cross correlation between price gap and inflation[1)] (simulation result under monetary targeting)



Notes: 1) Correlation coefficient between the price gap in period t and inflation(year-over-year) in period t+j

It is notable that the price gap calculated by using the conventional P-star model has little difference in terms of its predictive although it is somewhat different from the price gap defined in the theory of price-level determination. The usual P-star model estimates long-run equilibrium price by $\widetilde{P}_t^* = M_t \cdot V_t^*/Y_t^*$ ($\widetilde{P}_t^*$: P-star, V_t^*: long-run equilibrium velocity, Y_t^*: long-run equilibrium real GDP). Notably, P-star model calculates the long-run equilibrium price by estimating the trend of velocity. However, as can be seen in equation (32), the true long-run equilibrium price under monetary targeting, is affected by the growth rate of money in the previous period. Moreover, it is influenced by temporary shift in the preference for money. As the velocity in this paper's model has a constant value at the steady state, the price gap ($\log(\widetilde{P}_t^*/P_t)$) based on the P-star model is also computed by using this value as the trend of the velocity, it turns out that its predictive power for inflation (the panel (b) of <Figure 9>) is as high as that of the practical price gap (defined in the theory of price level determination) has.

C. Analysis of the Price Gap under the Interest Rate Rule

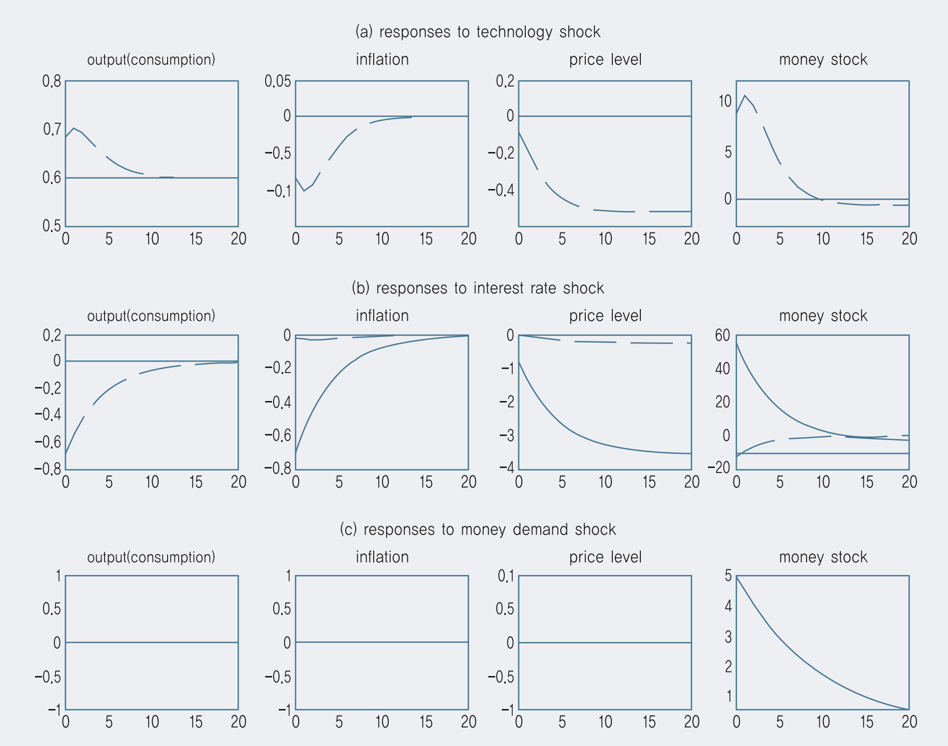
The impulse response functions under the interest rate rule are exhibited in <Figure 10>. First, when a positive technology shock occurs within an economy, potential output increases immediately and while actual output increases more than potential output and then declines. In other models, actual output following a technological shock gradually rises in contrast to the response of actual output in the model presented in this paper. This appears to stem from the fact that there are no habit formation in consumption and that interest rate smoothing is not considered. The long-run equilibrium price does not change because aggregate demand (consumption) and the aggregate supply (production capacity) increase on the same scale. Subsequently, the long-run equilibrium money stock (defined as the equilibrium money stock that appears when all the prices are flexible), increases in accordance with the degree of technological advance. However, the actual inflation rate declines thanks to the decrease in production cost driven by the technological advance but then returns to the targeted inflation rate set by the central bank. In this process, the actual price level drops permanently, exhibiting a discrepancy with the long-run equilibrium price. Looking into the response of actual money supply, it increases sharply in the short-run due to the increase in output and the reduction of the interest rate by the central bank, then declines reflecting a drop in the actual price level to a level where it remains[12] permanently lower than the long-run equilibrium price level.

When there is a contractionary interest shock, potential output does not change. However actual output returns to pre-shock level after initially decreasing. Under flexible prices, inflation falls quickly and then returns to the target inflation rate while, under sticky prices, inflation falls gradually only a bit and then returns to the target inflation rate. Due to the differences in the responses of inflation, the long-run equilibrium price and the actual price level exhibit a big discrephancy in the long-run. Meanwhile, as stated in Footnote 8, when prices are flexible, an interest rate shock acts rather as a factor that reduces the current nominal interest rate. As a result, the long-run equilibrium money stock sharply increases at an early stage, but later it rather decrease, reflecting a decline in the price level. In contrast, when prices are sticky, the money stock decreases in the short-term due to the falls in output, drops in the price level and increases in the nominal interest rate. Although both the actual money stock and the long-run equilibrium money stock decrease in the long-run, the long-run equilibrium money stock declines more steeply, reflecting the fact that the long-run equilibrium price declines more than the actual price level.

Finally, when a temporary money demand shock impinges on the economy, the central bank supplies money to accommodate money demand, and so there is no change in the interest rate. Consequently, the shock does not cause any

12) This dramatic increase in money stock stems from the fact that the interest rate elasticity of money demand in this model, $1/(R-1)$ is much larger than the interest rate elasticity empirically estimated. However, even if adjusting the interest rate elasticity of money demand more realistically, the implications gained from the long-run result of impulse responses are the same.

Figure 10 Impulse responses under the interest rate rule



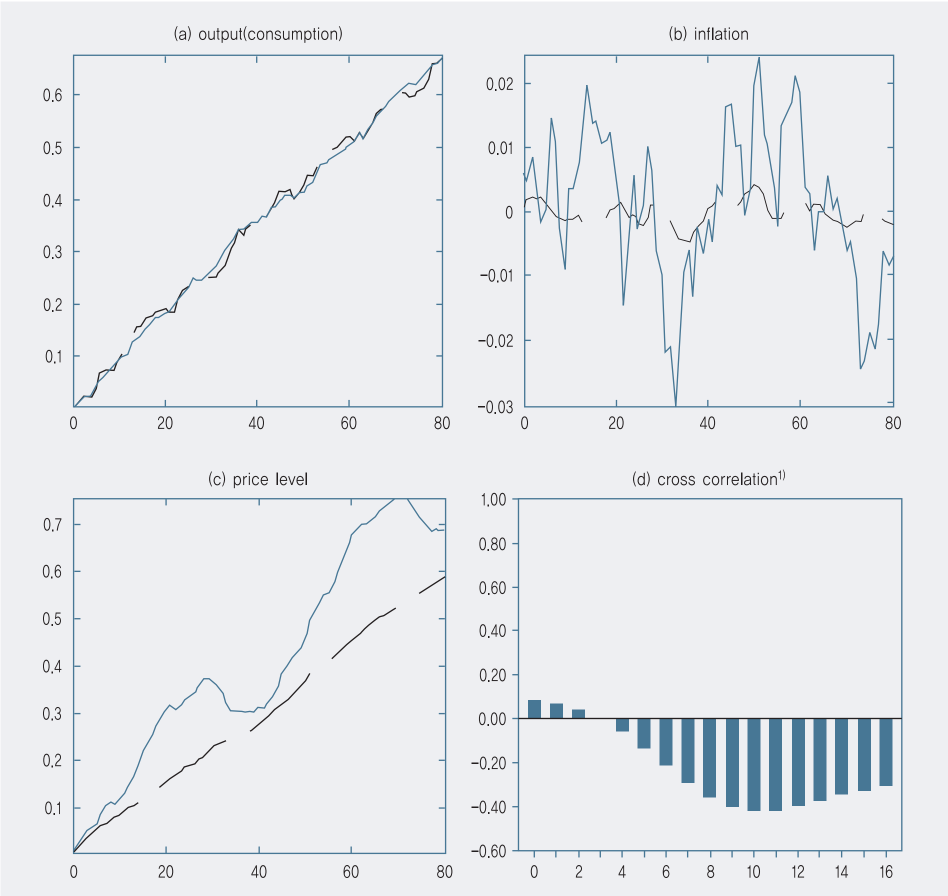
Notes: 1) Horizontal axes show quarters after a shock. Vertical axes represent percent changes for output, the price level and the money stock, and deviation from the steady state value for inflation.
2) Solid lines are responses when prices are flexible whereas dashed lines are responses when prices are sticky.

disruption to economic variables regardless of price rigidity with the exception of a temporary increase in the money stock. Thus, there is no discrepancy between the long-run equilibrium price and the actual price level.

I conduct a stochastic simulation in order to compare the movements of the long-run equilibrium price and that of actual price level in case that all the shocks in the economy occur consecutively. The results are displayed in <Figure 11>. Actual output goes up and down around potential output while the actual price level does not converge to the long-run equilibrium price. Moreover, in that the correlation between the price gap and future inflation generated by the simulation shows virtually opposite signs, the price gap does not predicts future inflation.

Putting all these discussions together, the actual price level converge to long-

Figure 11 **Simulation result under interest rate rule**



Notes: 1) Correlation coefficient between the price gap in period t and inflation (year-over-year) in period t+j
2) Horizontal axes show quarters after the initial period. Vertical axes represent log levels for output and the price level, and deviation from the steady state value for inflation.
3) Solid lines are responses when prices are flexible whereas dashed lines are responses when prices are sticky.

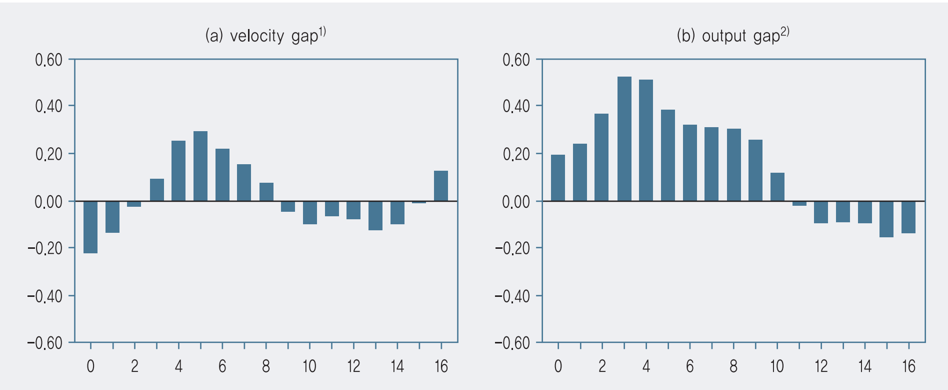
run equilibrium price under monetary targeting. This is because it is possible to control the price level eventually, if money is exogenously supplied under circumstances that the demand for money is stable. According to the simulation results from the models, the predictive power of the price gap for inflation has, is high and the its actual predictive power before the crisis reflects these theoretical results. However, under the interest rate rule that embeds the inflation targeting, although the shock to money demand does not generate a discrepancy between the actual price level (money stock) and the long-run equilibrium price (long-run money stock), the other shocks including the technology shock create the permanent gap between actual price level (money stock) and long-run equilibrium price (money stock). This is because when the shocks occur within an economy under inflation targeting, variations in the price level itself differs substantially depending on the pace at which the central bank returns the inflation rate to the target rate. Besides, since there is no variable acting as an anchor for the level of nominal variables as money is supplied endogenously, it is uncertain how the price level changes in the long-run. This implies that the method to predict the inflation by estimating long-run equilibrium price, is no more useful.

D. Evaluation of P-star Model

We have seen that the theoretical price gap has no power of predicting inflation under inflation targeting, but as can be seen in <Figure 2>, the price gap based on the P-star model still has some predictive power. In this section, I show that this is because the velocity gap and the output gap (GDP gap), which constitute the price gap, contain information about future inflation, not because the concept of the price gap is still useful.

That is, as $\widetilde{p}_t^* = m_t + v_t^* - y_t^*$, $p_t = m_t + v_t - y_t$ (all the variables are in logs), the price gap based on the P-star model can be expressed as:

Figure 12 Cross correlation between components of price gap and inflation



Note: 1) Correlation coefficient between the velocity gap in period t and GDP deflator inflation(year-over-year) in period t+j.

2) Correlation coefficient between the output gap in period t and GDP deflator inflation(year-over-year) in period t+j.

$$\underbrace{\widetilde{p}_t^* - p_t}_{price\ gap} = (\underbrace{v_t^* - v_t}_{velocity\ gap}) + (\underbrace{y_t - y_t^*}_{output\ gap}) \tag{48}$$

When velocity decreases thanks to the relative increase in the money stock compared to the size of the economy, with the velocity gap widens, if aggregate demand increases so actual output is greater than potential output, the output gap also widens. Both of these two factors act to expand the price gap. According to < Figure 12>, the output gap contains greater predictive power for inflation than the velocity gap during the period 2000:Q1~2008Q1. This result coincides with theories on the Phillips Curve. In this regard, the usefulness of the price gap as an indicator for predicting inflation comes mainly from the output gap. The velocity gap has relatively less predictive power for inflation than the output gap.

In discussions based on the theory of price level determination, the estimation of long-run equilibrium price under the interest rate rule is no longer useful in predicting inflation. However, the actual price gap estimated is still useful, which stems from the following conceptual difference. The long-run-equilibrium price can be approximately written as

$$p_t^* = m_t^* + v_t^* - y_t^* \tag{49}$$

where m_t^* is long-run equilibrium money stock.

This reflects the endogeneity of the money stock as money is supplied so as to accommodate the demand for money. Thus, the price gap is written as:

$$\underbrace{p_t^* - p_t}_{\substack{price\ gap \\ I(1)}} = (\underbrace{m_t^* - m_t}_{\substack{nominal\ money\ gap \\ I(1)}}) + (\underbrace{v_t^* - v_t}_{\substack{velocity\ gap \\ I(0)}}) + (\underbrace{y_t - y_t^*}_{\substack{output\ gap \\ I(0)}}) \tag{50}$$

As has already been mentioned in the impulse-response analysis, the responses of the long-run equilibrium money stock and the actual money stock to several structural shocks under the interest rate rule result in a permanent gap between the two in the long-run. In consequence, the price gap and the nominal money gap become non-stationary variables although the velocity gap and the output gap in the above equation are stationary variables, and the so price gap loses its predictive power for inflation. In contrast, the money stock actually observed to estimate the long-run equilibrium price in the P-star model. The price gap is reduced to a simple linear combination of the velocity gap and the output gap.

$$\underbrace{\widetilde{p}_t^* - p_t}_{\substack{\textit{price gap} \\ I(1)}} = \underbrace{(v_t^* - v_t)}_{\substack{\textit{velocity gap} \\ I(1)}} + \underbrace{(y_t - y_t^*)}_{\substack{\textit{output gap} \\ I(0)}}$$

Consequently, the price gap based on the P-star model is just a linear combination of information on the velocity gap and the output gap and not the one that is based on theoretical grounds. Not only that, the above equation has no micro-foundation to explain why the velocity and output gaps should be combined in a linear form. Even if the velocity and output gaps have predictive power for inflation, it is possible to consider generalized indicator for predicting inflation that could efficiently combine information on these two.[13] However, the P-star model does not provide answers for this.

V. Conclusions

In this paper, I examined what factors determine prices in the long-run under different monetary policy and analyzed whether it would be possible for the actual price level to converge to the long-run equilibrium price. The analytical results show that, under monetary targeting, the actual price level converges to the long-run equilibrium price that is determined by money supply. Consequently, under monetary targeting, the price gap based on the movement of money supply has high predictive power for inflation and its high predictive power before the currency crisis reflects these theoretical results.

However, under the interest rate rule that enables inflation targeting, the long-run equilibrium price level is determined by the interest rate, not by current money supply. Furthermore, when various types of shocks impinges on the economy, the actual price level does not converge to the long-run equilibrium level. This implies that predicting inflation by estimating the price gap and the long-run equilibrium price-level is no longer useful.

However, the above results show that there is still some correlation between the price gap based on the P-star model and inflation. This is mainly because of

13) For instance, it might be possible to consider a non-linear function $f(\overline{v}_t - v_t, v_t - \overline{v}_t)$ instead of simply adding the output gap and the velocity gap.

information on the output gap. Velocity gap has a relatively low predictive power for inflation. In this regard, methods other than the price gap model are necessary to predict inflation under inflation targeting. It would also be important to verify the usefulness of the velocity gap in predicting future inflation and its theoretical rationale. Although money is less likely to determine prices than before, it is possible that the velocity gap, which acts as a measure of excess liquidity. could offer useful information concerning the future economic situation.

An approach to explain this theoretically is to examine the process of portfolio adjustment. If there is no cost for portfolio adjustment, the relative increase in the money stock compared to the size of the economy could reflect fluctuations in the money preference for money of the private sector. In this case, as has been shown in the impulse-response analysis, the increase in the money stock relative to the size of the economy does not act as a factor that disrupts the economy.

However, if there is considerable costs associated with portfolio adjustments, excess liquidity in the current period can be interpreted as a result of incomplete portfolio adjustments in the past. In this case, excess liquidity in the current period may lead to massive portfolio adjustments in the future.

Under monetary targeting, even though there are portfolio adjustments, the central bank keeps money growth at a target rate. Thus prices should increase in line with the amount of increased money supply if the demand for real money balances is stable over time. However, under the interest rate rule money is supplied endogenously and the process of portfolio adjustments entails simultaneous adjustments of both money and prices. Even if the demand for real money balances is stable over time, future economic conditions may dictate the degree to which money and prices are adjusted. That is, the effects of portfolio on prices may differ depending on how money is spent, which is extremely uncertain to predict considering that money moves in quest of high yields and it depends on future economic conditions, Therefore, it would be difficult to find a stable relationship between the current money stock and future inflation.

In view of the fact that the effects of money on prices is uncertain, it is necessary to develop a variety of indicators and models and, at the same time, to improve the ability of interpreting information contained in the movements of the money stock in order to predict future inflation more accurately.

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