Breaking the Curse of Cash

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Abstract

Can eliminating paper money improve social welfare? We construct a dual currency model to study whether illegal activity can be reduced or eliminated by modifying the payment environment. In our model, there are two types of money, coins and paper money, and two types of goods, legal and illegal. Legal (goods) traders are *ex ante* indifferent between coins and paper money, but illegal (goods) traders prefer paper money to coins because illegal activities can be detected by the noise of the coins. Eliminating paper money can improve social welfare by reducing illegal activity. However, this pooling equilibrium is suboptimal when government finance is restricted. Given the government budget constraint, a separating equilibrium in which legal traders use coins and illegal traders use paper money, can improve welfare by making a transfer from the illegal traders to the legal traders.

Key Words: dual currency, seigniorage, externality
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1 Introduction

Despite the existence of alternative means of payment, including those that earn a higher rate of return, cash is still used in many transactions. One reason individuals might choose to use cash rather than other assets is that cash is free of record-keeping. For example, two parties to a transaction might prefer to exchange cash for goods and services to avoid the transaction costs associated with third-party processing. Alternatively, individuals who are engaged in illegal activity might also prefer cash because of the lack of record-keeping involved.

If illegal activity uses cash, and we want to discourage this activity, then a natural question is whether such activity can be reduced or eliminated by modifying the payment environment or altering transaction costs. Rogoff (2016) argues that policymakers should eliminate large denomination bills in order to reduce illegal transactions. Williamson (2017) agrees that the gain from reforming the currency system could be significant, but argues that there are two important costs associated with doing so. First, eliminating cash, specifically large denomination bills, can be costly to those who use these large denomination bills for legal purposes. If large denomination bills have been actively used in legal transactions, there is at least a proportion of people who choose to use these bills instead of other payment methods. So given elimination of large denomination bills, they would have to use an alternative payment method that is more costly. Second, the elimination of cash (or a specific bill) could result in a reduction of seigniorage to the central bank. The central bank can earn seigniorage by increasing the money supply to generate a strictly positive inflation rate. Moreover, raising the inflation rate reduces cash transactions as the rate of return on cash decreases. So eliminating cash, instead of raising the inflation rate, may not be the optimal solution when we consider the benefit of seigniorage to the government or central bank.

Rogoff (2016) claims that the first problem can be resolved by having the government provide some type of electronic money. In other words, if the cost of using cash sufficiently rises, then those who use cash for legal purchases will choose this newly provided electronic money. However, the welfare implications are unclear. For example, if the cost of using cash is high enough, legal traders will switch to another (previously more costly) payment method. While the reduction in illegal trade can provide a welfare benefit, legal traders will be worse off unless the cost of a newly introduced payment method is comparable to the cost previously associated with using cash. For the second problem, Rogoff (2016) argues that the loss of seigniorage revenues is likely canceled out by the benefits due to higher tax revenues from the underground economy. In his conjecture, tax evasion could be reduced when cash transactions can be recorded with electronic money. However, this argument depends on the degree of lost seigniorage relative to the decline in tax evasion. Furthermore, even if this sort of policy is welfare-enhancing, it is not obvious that this is the welfare-maximizing policy.

Given the potentially complex nature of the welfare implications of eliminating cash, the purpose of this paper is to use a model in which money is essential to examine payment choices when some trades are illegal. To do so, we use a modified version of the monetary search model of Rocheteau and Wright (2005), in which individuals differ *ex ante* in terms of their preferences. We assume that some fraction of agents engage in illegal trade. In reality, individuals engaged in illegal trade choose cash not only for the anonymity, but also because it can be spent in large denominations. As shown in Lee, Wallace, and Zhu (2005), the denomination structure of money is chosen to minimize holding or carrying costs of cash. In this respect our model addresses the issue related to denomination size by considering issues of portability.[1] In our model, we assume that there are two types of media of exchange: coins and paper money. Coins are costly to carry for illegal traders because they make noise. This can result in illegal activity being detected by some type of government authority who can punish the traders. Paper does not make a sound and therefore illegal trade cannot be detected if individuals use paper. Furthermore, we allow the exchange rate between coins and paper money to be determined by the market, rather than be arbitrarily fixed by the government.

The model produces a couple of important results. First, the model shows that eliminating paper money can improve welfare when there is a sufficiently large inefficiency in illegal trade.

[1]We do not discuss the optimal denomination structure, which we consider to beyond the scope of this paper.

In the model it is possible to eliminate paper money by increasing the cost of holding paper money. This does not eliminate illegal trade, but can reduce the amount of illegal trade.

Second, welfare is maximized in a separating equilibrium in which illegal traders use paper money and legal traders use coins. In the model, given limited government financing, there are 5 possible equilibria. Four of these equilibria are pooling equilibria in which both legal and illegal traders use at least one common media of exchange. The fifth equilibrium is the separating equilibrium in which the government can provide different rates of return on coins and paper money. If illegal traders impose an externality on society, the government can generate seigniorage from the illegal traders by setting low rate of return on paper money and providing transfers to legal traders by setting high rate of return on coins. Then the amount of illegal trade is reduced while the amount of legal trade increases. This is a standard solution to an externality problem.

2 Related Literature

One recent paper that studies the elimination of cash is Alvarez and Lippi (2017). They assess the welfare cost of phasing out cash in the context of a cash inventory model. In their model both cash and credit are simultaneously available and credit use depends on the level of cash holdings and a withdrawal cost. While they focus on the transformation from cash to credit for inhibiting illegal activities, we study a dual currency economy under lack of record-keeping technology.

After the seminal work of Kareken and Wallace (1981), multiple currencies have been studied in various types of theoretical models in which currency is essential for transactions and agents have a choice over which currencies to accept and use. In search theoretical models with indivisible money, multiple currencies can exist with different rate of returns since agents cannot access a centralized market to rebalance their portfolios. In most of these models multiple currencies do not lead to a welfare improvement mainly due to trade inefficiencies.[2]

[2]See Matsuyama et al. (1993), Trejos and Wright (1996), Zhou (1997), Ravikumar and Wallace (2002).

There are two counterexamples. Kiyotaki and Moore (2004) show that if sellers can produce either local goods or generic goods and the utility of local goods is greater than generic goods, then the equilibrium with local monies Pareto dominates the equilibrium with uniform money. In our paper we also have two types of goods, legal and illegal, and legal goods are more beneficial than illegal goods in terms of social welfare, but the types are fixed. So, unlike Kiyotaki and Moore (2004), the choice of payment methods can affect the quantity of production, but cannot change the types of goods produced in our model. In Kocherlakota and Krueger (1999), given a bias for home goods, separate monies signal the agent's preference and can be essential to achieve an optimal allocation. The idea of separating payment methods by preferences is similar to the approach in this paper, but we show that separate monies can be beneficial even with divisible money and a centralized market.

In the literature with divisible money, Zhang (2014) studies international currency competition and considers the role of seigniorage. Given a counterfeit risk associated with foreign currency and a national-currency-required transaction, currencies are treated asymmetrically by countries. If the currency is used as an international currency, the government considers seigniorage when implementing monetary policy. We also study multiple currency equilibria given heterogeneous preferences, multiple payment methods, and a government budget constraint. However, we focus more on welfare improvement by making transfers in a single economy rather than examining competition between countries.

Finally, this paper is also related to the literature that studies the societal benefits of illiquid assets. Specifically, the framework is close to Shi (2008), in which there are two goods with the same cost of production, but the marginal utilities differ. In his paper, a legal restriction on illiquid assets improves welfare by shifting production toward higher utility goods. In our paper, coins are more costly than paper money in illegal trade and this transaction cost associated with coins improves welfare by smoothing marginal utilities between the two goods. However, in our model the transaction cost is just used for revealing types instead of reducing the quantity of trade directly. So the result of smoothing marginal utilities comes from the seigniorage transfer between the two types.

3 Model

The model is an adaptation of Rocheteau and Wright (2005). Time is discrete and continues forever. Each time period is divided into two subperiods. In the first subperiod, agents interact in a centralized Walrasian market (CM). In the second subperiod, agents are matched pairwise to trade in a decentralized market (DM).

The model is populated by two types of individuals, buyers and sellers, who differ in terms of their preferences, as well as a government authority. Buyers want to consume in the DM and produce in the CM. Sellers want to consume in the CM and produce in the DM. There is a continuum with unit mass of each type. There is one good sold in the CM. There are two possible goods sold in the DM, legal and illegal goods. A fraction of buyers, ρ, only consume legal goods, whereas the remaining fraction, $1-\rho$, only consume illegal goods. Buyers know their own types in the CM and there is no asymmetric information.[3] Sellers are capable of producing both the legal and illegal good. For simplicity, we assume that sellers will produce whichever of the goods is demanded by the buyer. When buyers and sellers are matched in the DM, they negotiate the terms of trade. We assume that these meetings are anonymous in the sense that buyers and sellers do not know the trading histories of their trading partner. As a result, media of exchange are essential for trade. In the DM, buyers offer the media of exchange to sellers in exchange for goods. After leaving the DM, individuals enter the CM in the subsequent period where they can trade the CM good and reallocate their asset holdings.

Buyers have an expected lifetime utility of

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t^j)]$$

[3]We could avoid the ex ante heterogeneity of buyers by assuming that buyers know their types after the CM and providing an insurance contract for types. However, this ex ante heterogeneity is a simpler approach and the main results from the welfare analysis also do not change.

where $H_t \in \mathbb{R}$ is the supply of labor in the CM, x_t^j is the quantity of consumption in the DM, $j \in \{l, i\}$ denotes the individuals type with l used to denote a preference for legal goods and i used to denote a preference for illegal goods, $u(\cdot)$ is the utility generated from consumption in the DM where $u', -u'' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $-\frac{xu''(x)}{u'(x)} = \gamma \leq 1$. We assume that the buyers have a linear production technology such that one good is created using one hour of labor.

Sellers have an expected lifetime utility of

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t]$$

where X_t is consumption in the centralized market and h_t is hours worked in the decentralized market. Again, we assume that there is a linear production technology such that one hour of work produces one unit of the output good.

There are two assets that can be used as media of exchange, coins and paper money. Coins are "noisy" and therefore present a problem to illegal traders because they increase the likelihood of detection and punishment for such trade. However, buyers can pay a cost equal to a fixed percentage, α, of their coin offering to evade detection. Paper money is silent and therefore there is no way for anyone outside of the transaction to identify that illegal trade has taken place.

Finally, the government authority cannot collect taxes. As a result, government policy entails making lump sum transfers by issuing coins and paper money in CM and by conducting open market operations (using one type of money to purchase another). Let M_t denote the supply of coins and N_t denote the supply of paper money in the period t. Also, let ϕ_t denote the price of coins and ψ_t denote the price of paper money in terms of the CM good in period t. Suppose that at the beginning of time ($t = 0$), the government makes a transfer, τ_0, in the centralized market such that,

$$\tau_0 = \phi_0 M_0 + \psi_0 N_0$$

It follows that the government budget constraint for time $t = 1, 2, \ldots$ is given as

$$\tau_t = \underbrace{\phi_t(M_t - M_{t-1})}_{\text{Coin Seigniorage}} + \underbrace{\psi_t(N_t - N_{t-1})}_{\text{Paper Money Seigniorage}} \geq 0$$

where τ_t is the real value of the lump sum transfer given in period t. Given the government's inability to tax, we assume that τ_t must be non-negative. In what follows, we will assume that the initial transfer, τ_0, is sufficiently small to create a scarcity of assets in the model.

3.1 Kuhn-Tucker Conditions and Stationary Monetary Equilibria

In the model, each buyer knows their own type in the *CM*, so they choose their portfolio to maximize their expected trading gains in the *DM*. A buyer who consumes legal goods solves the following problem in the *CM* of period t:

$$\underset{m_t^l, n_t^l, x_t^l \geq 0}{Max} \; u(x_t^l) - m_t^l - n_t^l \tag{1}$$

subject to the seller's participation constraint,

$$\frac{\beta \phi_{t+1}}{\phi_t} m_t^l + \frac{\beta \psi_{t+1}}{\psi_t} n_t^l - x_t^l \geq 0. \tag{2}$$

The illegal buyer solves the following problem in the *CM* of period t:

$$\underset{m_t^i, n_t^i, x_t^i \geq 0}{Max} \; u(x_t^i) - (1+\alpha) m_t^i - n_t^i \tag{3}$$

subject to the seller's participation constraint,

$$\frac{\beta \phi_{t+1}}{\phi_t} m_t^i + \frac{\beta \psi_{t+1}}{\psi_t} n_t^i - x_t^i \geq 0. \tag{4}$$

All quantities in equations (1) through (4) are expressed in units of the *CM* consumption good in period t. In equations (1) through (4), m_t^l and n_t^l denotes the real quantities of coin and paper money held by the legal buyer while m_t^i and n_t^i denotes the quantities of coin and paper money held by the illegal buyer, respectively. Illegal buyers face a proportional transaction cost, $\alpha \in (0, 1)$, of using coins without creating a noise. This transaction cost allows the illegal trader to avoid detection. For example, illegal traders can rent a tinted minivan to carry coins secretly. Thus, this is not a cost of carrying heavy coins which would apply to everyone.[4]

From now on, we focus on stationary equilibria in which $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ and $\frac{\psi_{t+1}}{\psi_t} = \frac{1}{\eta}$ for all t. From the legal and illegal buyers' problems we have two Lagrangians, $\boldsymbol{L}^l$ and $\boldsymbol{L}^i$, which are written as

$$\boldsymbol{L}^l = u(x^l) - m^l - n^l + \lambda^l \left(\frac{\beta}{\mu} m^l + \frac{\beta}{\eta} n^l - x^l \right), \tag{5}$$

$$\boldsymbol{L}^i = u(x^i) - (1+\alpha) m^i - n^i + \lambda^i \left(\frac{\beta}{\mu} m^i + \frac{\beta}{\eta} n^i - x^i \right). \tag{6}$$

For each variable, m^l, n^l, m^i and n^i, the Kuhn-Tucker conditions are:

$$\begin{array}{llll}
(x^l) & u'(x^l) \leq \lambda^l; & x^l \geq 0; & x^l(u'(x^l) - \lambda^l) = 0, \\
(x^i) & u'(x^i) \leq \lambda^i; & x^i \geq 0; & x^i(u'(x^i) - \lambda^l) = 0, \\
(m^l) & \lambda^l \frac{\beta}{\mu} \leq 1; & m^l \geq 0; & m^l(\lambda^l \frac{\beta}{\mu} - 1) = 0, \\
(n^l) & \lambda^l \frac{\beta}{\eta} \leq 1; & n^l \geq 0; & n^l(\lambda^l \frac{\beta}{\eta} - 1) = 0, \\
(m^i) & \lambda^i \frac{\beta}{\mu} \leq 1+\alpha; & m^i \geq 0; & m^i(\lambda^i \frac{\beta}{\mu} - 1 - \alpha) = 0, \\
(n^i) & \lambda^i \frac{\beta}{\eta} \leq 1; & n^i \geq 0; & n^i(\lambda^i \frac{\beta}{\eta} - 1) = 0.
\end{array} \tag{7}$$

Note that if $\lambda^j = 0$ then $m^j = n^j = 0$ in the third column of equation (7), so that $x^j = 0$. Since we are interested in the equilibrium allocations which have strictly positive consumption, i.e. $x^j > 0$, we require $\lambda^j > 0$ for both $j = l$ and $j = i$.

[4] We can add a carrying cost to the model, but the main result will not change because only legal traders would become worse off with coins in either pooling or separating equilibrium.

Suppose $\lambda^j > 0$. We have the first-order conditions for x^l, x^i as

$$\begin{aligned} u'(x^l) &= \lambda^l, \\ u'(x^i) &= \lambda^i. \end{aligned} \tag{8}$$

Given the policy variables μ and η, we can write the different types of possible combinations of asset holdings for legal and illegal traders, respectively, as

$$\begin{aligned} m^l > 0, n^l = 0 \quad &\text{if} \quad \mu < \eta, \\ m^l = 0, n^l > 0 \quad &\text{if} \quad \mu > \eta, \\ m^l > 0 \quad \text{and} \quad n^l > 0 \quad &\text{if} \quad \mu = \eta. \end{aligned} \tag{9}$$

$$\begin{aligned} m^i > 0, n^i = 0 \quad &\text{if} \quad \mu(1+\alpha) < \eta, \\ m^i = 0, n^i > 0 \quad &\text{if} \quad \mu(1+\alpha) > \eta, \\ m^i > 0 \quad \text{and} \quad n^i > 0 \quad &\text{if} \quad \mu(1+\alpha) = \eta. \end{aligned} \tag{10}$$

Note that the choice of payment method depends on the relative rates of return on paper money and coins. If the rates of return on both paper and coins are equal, then the individual is indifferent between the two. If the rate of return on coins is higher than the rate of return on paper, then the individual will prefer to hold coins, and vice versa. Note, however, that the effective rate of return that an illegal trader receives on coins is lower than the rate of return for legal traders because legal traders do not have to pay a transaction cost to hide the noise of the coins. If buyers choose to use a particular method of payment then the corresponding first column of equation (7) will hold with equality. So the consumption levels of the buyers, x^l, x^i, are determined by the first column of equation (7) and equation (8) given the monetary policy μ and η. Then the demands for paper money and coin money can be derived from the binding constraints, equations (2) and (4).

In equilibrium, asset markets clear in the *CM* for all t, so that the total demand for coins and paper money are equal to the supply of outstanding coins and paper money, respectively, as

$$\begin{aligned} \rho m^l + (1-\rho) m^i &= \bar{m}, \\ \rho n^l + (1-\rho) n^i &= \bar{n}, \end{aligned} \tag{11}$$

where $\bar{m}, \bar{n}$ are the real supply of paper money and coin money in equilibrium. Note that in stationary equilibrium the real money supplies are constant over time as $\bar{m} := \phi_t M_t = \phi_{t-1} M_{t-1}; \bar{n} := \psi_t N_t = \psi_{t-1} N_{t-1}$, so that the government budget constraint is written as

$$\tau = \left(1 - \frac{1}{\mu}\right) \{\rho m^l + (1-\rho) m^i\} + \left(1 - \frac{1}{\eta}\right) \{\rho n^l + (1-\rho) n^i\} \geq 0. \tag{12}$$

Definition 1. *Given a monetary policy (μ, η), a stationary monetary equilibrium consists of the variables $(m^l, n^l, x^l, m^i, n^i, x^i)$ and the multipliers λ^l, λ^i and the transfer τ, which satisfy the binding constraints (2) and (4), the Kuhn-Tucker conditions in equation (7), the first-order conditions in equation (8), market clearing conditions, equation (11), and the government budget constraint, equation (12).*

Finally, by adding expected utilities across agents in a stationary equilibrium, we can write the welfare function as

$$W = \rho\{u(x^l) - x^l\} + k(1-\rho)\{u(x^i) - x^i\} - c(\alpha), \tag{13}$$

where $c(\alpha) = \alpha m^i$ is the transaction cost of using coins and $-\infty < k < 1$ denotes the external inefficiency associated with illegal trades.

3.2 Types of Equilibria

Given the equilibrium conditions, there are five different equilibrium cases which are feasible for a monetary policy (μ, η). To see this, consider Table 1.

In the table, an 'X' denotes an equilibrium that is not possible whereas a '+' denotes an equilibrium that is possible. Which equilibria are possible depends on the relative rates of return on coins and paper. Note that there is only one separating equilibrium and that is the

Table 1: Possible Equilibria

	Legal	Coins only ($\mu < \eta$)	Paper only ($\mu > \eta$)	Coins and paper ($\mu = \eta$)
Illegal	Coins only ($\mu(1+\alpha) < \eta$)	+	X	X
	Paper only ($\mu(1+\alpha) > \eta$)	+	+	+
	Coins and paper ($\mu(1+\alpha) = \eta$)	+	X	X

case in which illegal traders only hold paper money and legal traders only hold coins. In all other equilibria, one of the media of exchange is held by both types.

3.2.1 Pooling equilibrium with paper money ($\mu > \eta \geq 1$)

If $\mu > \eta$, then both legal and illegal buyers will use paper money and $n^l, n^i > 0$ and $m^l = m^i = 0$ in equilibrium. Since this is a pooling equilibrium with paper money only, there is no exchange rate between paper money and coin money. The rate of return for using paper money, i.e. $\frac{1}{\eta}$, is same for both types of buyers, so $x^l = x^i$ in equilibrium and there is no punishment of illegal trades since everyone uses paper money and paper money cannot be detected. Note that the government budget constraint requires $\eta \geq 1$, since coins are not held in equilibrium. This equilibrium case is shown as the area of ① in Figure 1.

[Figure 1 here]

3.2.2 Pooling equilibrium with coins ($\frac{\eta}{1+\alpha} > \mu \geq 1$)

If $\frac{\eta}{1+\alpha} > \mu$, then both legal and illegal buyers use coins and $m^l, m^i > 0$ and $n^l = n^i = 0$ in equilibrium. Since this is a pooling equilibrium with only coin money, there is no exchange rate between paper money and coin money. However, the rate of return for using coin money is different by the types of the buyers, so given $\frac{\eta}{1+\alpha} > \mu$ we have $x^l > x^i$ in equilibrium. Since $m^i > 0$ in equilibrium the resource cost for transaction, $c(\alpha)$ is strictly positive in equilibrium. Note that since nobody holds paper money in this equilibrium, the government budget constraint also requires $\mu \geq 1$. This equilibrium case is shown as the area of ② in Figure 1.

3.2.3 Partially pooling equilibrium with paper money ($\mu = \eta \geq 1$)

If $\mu = \eta$, legal buyers are indifferent between using paper money or coins, but illegal buyers will only use paper money, so that $m^l, n^l, n^i > 0$ and $m^i = 0$ in equilibrium. Legal buyers are indifferent between holding either form of money because they each provide the same rate of return. However, illegal buyers will not hold coins because they face the added cost of hiding these coins. It follows from equations (7) and (8) that $x^l = x^i$ in equilibrium. Moreover, if both monies are actively used in equilibrium there is an indeterminacy in the exchange rate between paper money and coin money because it can be arbitrarily chosen by the government. This indeterminacy in the relative price is the same result from Kareken and Wallace (1980). In the initial period a price pair (ϕ_0, ψ_0) is arbitrarily determined by the initial supply of monies, M_0, N_0, and then the exchange rate is constant in the following periods. Although both types of money are used, the government budget constraint still requires $\eta \geq 1$ regardless of the real money demands, m^l, n^l, and n^i, because the rates of return on paper money and coins are the same. This equilibrium case is shown as all points along line ③ in Figure 1.

3.2.4 Partially pooling equilibrium with coins ($\frac{\eta}{1+\alpha} = \mu \geq \bar{\mu}$)

If $\frac{\eta}{1+\alpha} = \mu$, then illegal buyers are indifferent between paper money and coins, but legal buyers prefer to use coins, so that $m^i, n^i, m^l > 0$ and $n^l = 0$ in equilibrium. Furthermore, it follows that $x^l > x^i$. Since the rates of return for using two monies are equal for illegal buyers, there is also an indeterminacy in the exchange rate between two monies. Moreover, the exchange rate, i.e. the price of coin money divided by the price of paper money, increases over time by $1 + \alpha$. So this equilibrium case is shown as line ④ in Figure 1. Consider the case in which he government budget constraint (12) binds. If $\mu \geq 1$ then equation (12) does not bind. However, if $\mu < 1$ then equation (12) could bind according to the amount of m^i, n^i because a strictly positive amount of n^i with $\eta > 1$ is required to satisfy (12). That means, a proportion of illegal traders, $\theta \in (0, 1]$, must use paper money to satisfy the government budget constraint (12). Given $\frac{\eta}{1+\alpha} = \mu < 1$, the threshold θ^* can be obtained as

$$\theta \geq \theta^* = \left(\frac{1-\mu}{(1+\alpha)\mu - \mu}\right)\left(1 + \frac{f(\frac{\mu}{\beta})}{f(\frac{(1+\alpha)\mu}{\beta})}\right). \tag{14}$$

by plugging the first-order conditions, i.e. $u^{'}(x^l) = \frac{\mu}{\beta}$, $u^{'}(x^i) = \frac{\eta}{\beta}$, and the binding collateral constraints, i.e. $\frac{\beta}{\mu}m^l = x^l$, $\frac{\beta}{\mu}m^i = (1-\theta)x^i$, and $\frac{\beta}{\eta}n^i = \theta x^i$ into (12). We define $f(\cdot)$ as the inverse function of $u^{'}(\cdot)$ which is strictly decreasing and strictly convex. Note that given the assumption of $-\frac{xu''(x)}{u'(x)} = \gamma \leq 1$, θ^* is strictly decreasing in μ because the term in the second parenthesis of (14) is constant in μ.[5]. If $\mu = 1$, then $\theta^* = 0$ in (14). When μ falls, θ^* also decreases and there is a lower bound of μ, $\bar{\mu}$, at which $\theta^* = 1$ with $m^i = 0$ in equilibrium. Note that the allocation at $\mu = \bar{\mu}$, illustrated as the point C in Figure 1, is a separating equilibrium since $m^l, n^i > 0, m^i = 0$. When we define $\bar{\eta}$ as $\bar{\eta} = \bar{\mu}(1+\alpha)$, $\bar{\eta} > 1$ holds in (12) because $m^i = 0$ at $\mu = \bar{\mu} < 1$.

3.2.5 Separating equilibrium ($(1+\alpha)\mu > \eta > \mu \geq \bar{\mu}$)

If $(1+\alpha)\mu > \eta > \mu$, legal buyers prefer to use coins, but illegal buyers prefer to use paper money, so $m^l, n^i > 0$ and $n^l = m^i = 0$ in equilibrium and $x^l > x^i$ because $\eta > \mu$. This is a separating equilibrium, but since both monies circulate we have an exchange rate between paper money and coin money. The initial exchange rate can be chosen arbitrarily, but the growth rate of the exchange rate can be determined between 1 and $1+\alpha$. Using the first-order conditions, $u^{'}(x^l) = \frac{\mu}{\beta}$, $u^{'}(x^i) = \frac{\eta}{\beta}$, and the binding collateral constraints, $\frac{\beta}{\mu}m^l = x^l$, $\frac{\beta}{\eta}n^i = x^i$, we can re-write the government budget constraint with policy variables, μ and η, given in equation (12) as

$$\tau = \rho\left(\frac{\mu-1}{\beta}\right)f\left(\frac{\mu}{\beta}\right) + (1-\rho)\left(\frac{\eta-1}{\beta}\right)f\left(\frac{\eta}{\beta}\right) = 0, \tag{15}$$

[5]If $u^{'}(\cdot)$ is homogeneous of degree $-\gamma$, then $f(\cdot)$ is homogeneous of degree $-\frac{1}{\gamma}$

and shown as the GBC(SE) curve in Figure 1.[6] Note that the binding government budget constraint (14) intersects not only the point A with $\mu = \eta = 1$, but also the point C with $\mu = \bar{\mu}, \eta = \bar{\eta}$ in Figure 1 where $(\bar{\mu}, \bar{\eta})$ satisfies with $\bar{\eta} = (1+\alpha)\bar{\mu}$ and Eq. (14). The separating equilibrium, like the partially pooling equilibrium with coins allows for $\mu < 1$ as long as $\eta > 1$ because each money is demanded by separated parties, legal and illegal buyers. Thus, there is flexibility in the choice of the two monetary policy variables, μ and η, such that policy can actively influence the equilibrium allocation in this case. Moreover, the feasible set of monetary policy variables with two monies is expanded in the separating equilibrium to include the area of the triangle ABC. In the model issuing two monies expands the feasible allocation set because two monies not only separates the types, but also makes a transfer from one type to the other possible by using different rates of return. This equilibrium case is shown as the area of ⑤ in Figure 1.

4 Welfare Analysis

4.1 Optimal Policy

In this subsection we compare the welfare of the possible equilibria and try to determine the optimal monetary policy. In order to have a feasible policy solution, we need the assumption on the degree of the external inefficiency, k, in the welfare function as $k \geq -\frac{\rho}{1-\rho}$. For example, if $k < -\frac{\rho}{1-\rho}$ then the optimal policy in the equilibrium ① is $\mu^* = \eta^* = \infty$, in which autarky is better than any monetary equilibrium for society.

Lemma 1. *If $-\frac{\rho}{1-\rho} < k < 1$, reducing μ or η in pooling equilibrium improves welfare.*

[6] In order to guarantee the negative relationship between μ and η in (15), we assume that α is sufficiently small to satisfy $\hat{\eta} = 1 + \alpha < \frac{1}{1-\gamma}$ where $-\frac{xu^{''}(x)}{u^{'}(x)} = \gamma$ in the model. Since the seigniorage from the illegal traders, $(1-\frac{1}{\eta})(1-\rho)n^i = (1-\rho)\{(1-\gamma)u(x^i) - \frac{1}{\beta}x^i\}$, is a kind of laffer curve, it is maximized at $\eta = \frac{1}{1-\gamma}$ because of $u^{'}(x^i) = \frac{\eta}{\beta} = \frac{1}{\beta(1-\gamma)}$.

Proof. *Since* $x^l = x^i$ *from* $u^{'}(x^l) = u^{'}(x^i) = \frac{\eta}{\beta}$ *in the equilibrium cases* ①-②*, the welfare effect is*

$$\begin{aligned} \frac{\partial W}{\partial \eta} &= \rho\{u^{'}(x^l) - 1\}\frac{\partial x^l}{\partial \eta} + k(1-\rho)\{u^{'}(x^i) - 1\}\frac{\partial x^i}{\partial \eta} \\ &= \{\rho + k(1-\rho)\}\{u^{'}(x^l) - 1\}\frac{\partial x^l}{\partial \eta}. \end{aligned} \tag{16}$$

where $\frac{\partial x^l}{\partial \eta} = \frac{1}{\beta u^{''}(x^l)} < 0$*. Thus,* $\frac{\partial W}{\partial \eta} \leq 0$ *as long as* $k \geq -\frac{\rho}{1-\rho}$*. Similarly, we have* $x^l > x^i$ *from* $u^{'}(x^l) = \frac{\mu}{\beta}, u^{'}(x^i) = \frac{(1+\alpha)\mu}{\beta}$ *in the equilibrium cases* ③-④*. The welfare effect is*

$$\begin{aligned} \frac{\partial W}{\partial \mu} &= \rho\{u^{'}(x^l) - 1\}\frac{\partial x^l}{\partial \mu} + k(1-\rho)\{u^{'}(x^i) - 1\}\frac{\partial x^i}{\partial \mu} \\ &= \frac{\rho}{\beta}\frac{\{u^{'}(x^l)-1\}}{u^{''}(x^l)} + \frac{k(1-\rho)}{\beta}\frac{(1-\alpha)\{u^{'}(x^i)-1\}}{u^{''}(x^i)}. \end{aligned} \tag{17}$$

Suppose that $u(x)$ *is a constant relative risk aversion(CRRA) utility function with* γ *as* $-\frac{xu^{''}(x)}{u^{'}(x)} = \gamma$ *and* $xu^{'}(x)$ *is strictly increasing. Then from* $u^{'}(x^l) = (1+\alpha)u^{'}(x^i)$ *and* $x^l > x^i$*, we know that* $\frac{\{u^{'}(x^l)-1\}}{u^{''}(x^l)} < \frac{(1-\alpha)\{u^{'}(x^i)-1\}}{u^{''}(x^i)}$ *holds. Thus* $\frac{\partial W}{\partial \mu} < 0$ *as long as* $k \geq -\frac{\rho}{1-\rho}$*. QED*

Lemma 2. *The welfare of the equilibrium allocation at* $\mu > 1, \eta = 1$ *is the same as the one at* $\mu = \eta = 1$ *while the welfare of equilibrium allocation at* $\mu = 1, \eta > \hat{\eta}$ *is the same as the one at* $\mu = 1, \eta = \hat{\eta}$*.*

Proof. *In equilibrium with* $\mu > 1, \eta = 1$ *coin money is not used, thus* μ *is irrelevant. In equilibrium with* $\mu = \eta = 1$ *coin money can be used along with paper money, but the allocation will not change because coin money can be replaced by paper money given the same rate of return. Similar logic applies for the case of* $\mu = 1, \eta > \hat{\eta}$*. QED*

Lemma 1 and 2 show that welfare is maximized at the point A in Figure 1 with $\mu = \eta = 1$ among the pooling equilibrium allocations in the area ① and ③ whereas welfare is maximized at the point B in Figure 1 with $\frac{\eta}{1+\alpha} = \mu = 1$ among the pooling equilibrium allocations in the area ② and ④.

Proposition 1. *If* $-\frac{\rho}{1-\rho} < k < 0$*, the welfare of the equilibrium allocation at* $\mu = 1, \eta = \hat{\eta}$ *is greater than the one at* $\mu = \eta = 1$ *whereas if* $0 < k < 1$*, the welfare of the equilibrium allocation at*

$\mu = 1, \eta = \hat{\eta}$ *is smaller than the one at* $\mu = \eta = 1$.

Proof. *Since* $\hat{\eta} > 1$, x^i *at* $\mu = 1, \eta = \hat{\eta}$ *equilibrium is less than* x^i *at* $\mu = \eta = 1$ *equilibrium while* x^l *is the same for both equilibrium cases. Thus it is straight-forward by the definition of welfare function.QED*

Proposition 1 confirms the intuition of Rogoff (2016) that if the external inefficiency of illegal trade is sufficiently large then eliminating paper money can improve welfare. However, notice that the welfare improvement depends on the degree of the external inefficiency. Thus eliminating paper money may worsen the allocation if the external inefficiency is not sufficiently large.

Lemma 3. *For given a* η, *reducing* μ *in separating equilibrium improves welfare.*

Proof. *Since* x^l *is independently determined from* $u^{'}(x^l) = \frac{\mu}{\beta}$ *in the separating equilibrium case* ⑤, *the welfare effect is*

$$\frac{\partial W}{\partial \mu} \quad = \quad \rho\{u^{'}(x^l) - 1\}\frac{\partial x^l}{\partial \mu} = \frac{\rho\{u^{'}(x^l)-1\}}{\beta u^{''}(x^l)} \leq 0.\textit{QED} \tag{18}$$

From Lemma 3 we know that the separating equilibrium allocations with $\eta \geq \hat{\eta}$ are suboptimal to the pooling equilibrium allocations on line ④ and so the allocation at the point B in Figure 1. Similarly, the separating equilibrium allocations with $\eta < \hat{\eta}$ can be maximized at the allocations on lines BC and AC in the Figure 1. So we can compare the welfare of separating equilibrium allocations on lines BC and AC along with the pooling equilibrium allocations at the point A and B.

Lemma 4. *For given* μ, *if* $k < 0$, *raising* η *in separating equilibrium improves welfare. If* $k > 0$, *reducing* η *in separating equilibrium improves welfare.*

Proof. *Since* x^i *is independently determined from* $u^{'}(x^i) = \frac{\eta}{\beta}$ *in the separating equilibrium case* ⑤, *the welfare effect is*

$$\frac{\partial W}{\partial \eta} \;=\; k(1-\rho)\{u^{'}(x^i)-1\}\frac{\partial x^i}{\partial \eta} = \frac{k(1-\rho)\{u^{'}(x^i)-1\}}{\beta u^{''}(x^i)}. \tag{19}$$

So if $k<0$ *then* $\frac{\partial W}{\partial \eta}>0$ *whereas if* $k>0$, $\frac{\partial W}{\partial \eta}<0$.*QED*

From Proposition 1 and Lemma 4, we know that if $-\frac{\rho}{1-\rho}<k<0$ then we need to compare the welfare of pooling equilibrium at $\mu=1,\eta=\hat{\eta}$ with the one of separating equilibrium in the line BC while if $0<k<1$, we need to compare the welfare of the pooling equilibrium at $\mu=\eta=1$ with the separating equilibrium on line AC.

Proposition 2. *There exists a separating equilibrium that provides greater welfare than the pooling equilibria.*

Proof. *Suppose* $-\frac{\rho}{1-\rho}<k<0$. *Our claim is that reducing* μ *along the line BC can improve welfare at the equilibrium allocation with* $\mu=1,\eta=\hat{\eta}$. *The logic is the same as the latter part in the proof of Lemma 1. Since* $u^{'}(x^l)=\frac{\mu}{\beta}, u^{'}(x^i)=\frac{(1+\alpha)\mu}{\beta}$ *holds along with the line BC, the welfare effect is*

$$\begin{aligned}\frac{\partial W}{\partial \mu} \;&=\; \rho\{u^{'}(x^l)-1\}\frac{\partial x^l}{\partial \mu}+k(1-\rho)\{u^{'}(x^i)-1\}\frac{\partial x^i}{\partial \mu}\\ &=\; \frac{\rho}{\beta}\frac{\{u^{'}(x^l)-1\}}{u^{''}(x^l)}+\frac{k(1-\rho)}{\beta}\frac{(1-\alpha)\{u^{'}(x^i)-1\}}{u^{''}(x^i)}.\end{aligned} \tag{20}$$

Given the assumptions that $-\frac{xu^{''}(x)}{u^{'}(x)}=\gamma$ *and* $xu^{'}(x)$ *is strictly increasing,* $\frac{\{u^{'}(x^l)-1\}}{u^{''}(x^l)}<\frac{(1-\alpha)\{u^{'}(x^i)-1\}}{u^{''}(x^i)}$ *holds because* $u^{'}(x^l)=(1+\alpha)u^{'}(x^i)$ *and* $x^l>x^i$. *Thus* $\frac{\partial W}{\partial \mu}<0$ *because* $k\geq-\frac{\rho}{1-\rho}$ *by assumption. Now suppose* $0<k<1$. *Our claim is that reducing* μ *along the line AC can improve welfare at the equilibrium allocation with* $\mu=\eta=1$. *Note that the government budget constraint binds along line AC as*

$$\begin{aligned}\tau \;&=\; (1-\tfrac{1}{\mu})\rho m^l+(1-\tfrac{1}{\eta})(1-\rho)n^i\\ &=\; \rho\{u^{'}(x^l)-\tfrac{1}{\beta}\}x^l+(1-\rho)\{u^{'}(x^i)-\tfrac{1}{\beta}\}x^i=0,\end{aligned} \tag{21}$$

and we have $\frac{\partial x^i}{\partial x^l}=-\frac{\rho}{1-\rho}$ *at the equilibrium allocation with* $\mu=\eta=1$ *from Eq. (20). So the* *welfare effect is*

$$\begin{aligned}\frac{\partial W}{\partial \mu} &= \rho\{u'(x^l)-1\}\frac{\partial x^l}{\partial \mu}+k(1-\rho)\{u'(x^i)-1\}\frac{\partial x^i}{\partial x^l}\frac{\partial x^l}{\partial \mu} \\ &= \rho\{u'(x^l)-1\}\frac{\partial x^l}{\partial \mu}-k\rho\{u'(x^i)-1\}\frac{\partial x^l}{\partial \mu},\end{aligned} \tag{22}$$

where** $\frac{\partial x^l}{\partial \mu}=\frac{1}{\beta u''(x^l)}<0$. **Since** $x^l=x^i$ **in the equilibrium allocation with** $\mu=\eta=1$ **and** $k<1$, $\frac{\partial W}{\partial \mu}<0$. **Finally, if** $k=0$, **the optimal monetary policy is** $\mu=\bar{\mu}$ **since** x^l **is maximized at the point C in the Figure 2.QED

[Figure 2 here]

Proposition 2 shows that there is a welfare-improving separating equilibrium in either case of $k<0$ or $k>0$. This welfare improvement comes from the expansion of the feasible allocation set as shown in Figure 2. The triangle area ABC is infeasible under pooling equilibrium, but feasible under separating equilibrium. This result is because separating types of buyers with two different payment methods allow the government to collect seigniorage from one type and make a transfer to the other type. So if there is an externality to adjust in the economy, separating types by providing two monies can be optimal because it can align the marginal rate of substitution between types to the social preference. For instance, if the inefficiency of illegal trade is not too large with $k>0$, the social welfare indifference curve has a tangency on line AC. In this case the government budget constraint binds because illegal trade also provides a positive marginal utility. On the other hand, if the inefficiency of illegal trade is sufficiently large with $k<0$, then the social welfare indifference curve has a tangency in the line of BC. In this case, the government budget constraint does not need to bind because reducing the illegal trade is optimal for society.

Notice that a separating equilibrium can exist for any $\alpha>0$, so that the main result of the paper, Proposition 2, does not depend on the size of this transaction cost. Furthermore, this transaction cost is not a resource cost wasted in optimal allocations, because in a separating equilibrium illegal traders use paper money and legal traders use coins at no cost.

4.2 Discussion

We have several points to discuss about the results of the model. First of all, in a separating equilibrium currency choices reveal types and allow the government to make a transfer from those engaging in illegal trade to those engaged in legal trade. So the government can improve welfare by adjusting the exchange rate between the two monies. In this paper legal trade is socially desirable while illegal activities are not. Thus, given heterogeneous preferences over payment methods, collecting more taxes from illegal traders and providing a transfer to legal trades is beneficial. A practical implementation of this plan would be to allow for two types of money to circulate (e.g., currency and e-money), allow the exchange rate between these two types of money to fluctuate, and use monetary policy to facilitate the transfer described in our model.

Second, the set of pooling equilibrium allocations is nested inside the set of separating equilibrium allocations. For example, if $k = 1$ then the optimal monetary policy is described as the point A in Figure 2. This equilibrium allocation can be generated either by one paper money or both paper money and coins with the same rates of return. Unlike most dual currency models with indivisible money, in this model there is a desired equilibrium allocation that dual currencies can achieve that uniform currency cannot. One reason for this result is that the framework with divisible money and a competitive market focuses on the intensive margin of trade rather than the extensive margin. Thus, search and/or matching efficiencies are not considered in our model and results.

Finally, in this type of model money is neutral, but not super-neutral because money is divisible and the rate of return on money determines the type of money and the quantity of trade. Similarly, the exchange rate between coins and paper money in separating equilibrium is also neutral, but not super-neutral. So the initial level of exchange rate is indeterminate as shown in Kareken and Wallace (1981), but the growth rate of exchange rate can be a policy variable to affect equilibrium allocations. If nominal interest payments on money are feasible, as shown in Andolfatto (2010), then a strictly positive nominal interest rate on coins can support the same equilibrium allocation instead of changing the exchange rate between paper money and coins.

5 The Role of the Government Budget Constraint

In this section we consider the role of the government budget constraint in generating our results. We do so first by considering the model without a government budget constraint. We then consider how the results would change if a lump-sum tax cannot be enforced.

5.1 The Model Without the Government Budget Constraint

The government budget constraint is the key assumption that enables us to have a welfare-improving separating equilibrium. Without the government budget constraint (12), only no arbitrage conditions $\mu \geq \beta$, $\eta \geq \beta$ are required for the feasible monetary policy (μ, η) in equilibrium. As shown in Figure 3, there are still five different equilibrium cases. For the welfare analysis, since Lemma 1-3 still apply, we can simply compare the welfare associated with points D and F and the line DF in Figure 3. Given $\mu = \beta$ when η increases from β to $(1-\alpha)\beta$, $x^l = x^*$ is fixed while x^i decreases. If $0 < k < 1$, welfare is maximized at the point F whereas if $k < 0$, then welfare is maximized at the point D. If $k = 0$, then the public is indifferent between line DF and points D and F. Therefore, without the government budget constraint, a separating equilibrium cannot improve welfare: Eliminating paper money, $\eta > (1-\alpha)\beta$, is optimal when the external inefficiency of illegal trade is sufficiently large ($k < 0$).

[Figure 3 here]

5.2 An Endogenous Government Budget Constraint

In this subsection we consider whether a separating equilibrium can still maximize welfare if agents refuse to pay the lump-sum tax. We describe the feasible consumption levels (x^l, x^i) for pooling equilibria with paper money, coins, and the separating equilibrium.

5.2.1 Pooling equilibrium with paper money

For type j agents,

$$\tau - n^j + u(x^j) \geq 0, \tag{23}$$

is required to pay the (negative) lump-sum transfer voluntarily in the period t CM. Otherwise, the type j agents would opt out in favor of autarky. Since the transfer, τ, must be supported by seigniorage,

$$\tau = \left(1 - \frac{1}{\eta}\right)\{\rho n^l + (1-\rho)n^i\}, \tag{24}$$

we can transform (23) into $-\frac{1}{\eta}n^j + u(x^j) \geq 0$. Note that $n^l = n^i$ holds in (24), because the equilibrium conditions are same for both types. By plugging the binding constraint, $\frac{\beta}{\eta}n^j = x^j$, into (24), we can have $x^j\{u^{'}(x^j) - \frac{1}{\beta}\} + u(x^j) - x^j u^{'}(x^j) \geq 0$, which can be reduced to

$$\beta u(x^j) \geq x^j. \tag{25}$$

Notice that $x^j\{u^{'}(x^j) - \frac{1}{\beta}\}$ and $u(x^j) - x^j u^{'}(x^j)$ represent the seigniorage from both types and the trade gain of both types, respectively. Thus, the incentive constraint (25) implies that the trading gain must be greater than the lump-sum tax. Define $\tilde{x}$ as x^j which satisfies (25) with equality. If β is sufficiently small, then $\tilde{x} < x^*$, thus the monetary policy could be limited as $\eta \geq \tilde{\eta}$ where $u^{'}(\tilde{x}) = \frac{\tilde{\eta}}{\beta}$. Note that $\tilde{\eta} < 1$: $\beta u^{'}(\tilde{x}) < 1$ holds because $\beta u(x) - x$ is decreasing at $x = \tilde{x}$. We can also confirm $\tilde{\eta} < 1$ with a negative transfer in equilibrium, $\tau = n^j - u(x^j) = \tilde{x}u^{'}(\tilde{x}) - u(\tilde{x}) = \tilde{x}u^{'}(\tilde{x}) - \frac{1}{\beta}\tilde{x} < 0$ in (23).

5.2.2 Pooling equilibrium with coins

This case is similar to the previous case, but $m^i < m^l$ because of $u^{'}(x^i) = \frac{(1+\alpha)\mu}{\beta}$. So the tax incentive constraint only binds for type i as

$$\tau - (1+\alpha)m^i + u(x^i) \geq 0. \tag{26}$$

By plugging $\tau = (1-\frac{1}{\mu})\{\rho m^l+(1-\rho)m^i\}$ and the binding constraint, $\frac{\beta}{\mu}m^j = x^j$ into (26), we can rewrite it as $\{\rho x^l+(1-\rho)x^i\}\{u^{'}(x^l)-\frac{1}{\beta}\}+u(x^i)-x^iu^{'}(x^i) \geq 0$. By using $u^{'}(x^i) = (1+\alpha)u^{'}(x^l)$ and assuming $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ we can transform (27) into

$$(1-\gamma+\frac{\gamma}{\delta})\beta u(x^j) \geq x^j, \tag{27}$$

where we define $\delta = \frac{\rho(1+\alpha)^{\frac{1}{\gamma}}+1-\rho}{1+\alpha}$. If we define $\hat{x}$ as x^i which satisfies (27) with equality, then note that $\hat{x} < \tilde{x}$, so that the lower bound $\hat{\mu}$, where $u^{'}(\hat{x}) = \frac{\hat{\mu}}{\beta}$ holds, is smaller than $\tilde{\eta}$.

5.2.3 Separating equilibrium

In separating equilibrium we need to consider three tax incentive constraints,

$$\begin{aligned} \tau - m^l + u(x^l) &\geq 0, \\ \tau - n^i + u(x^i) &\geq 0, \\ \tau - n^i + u(x^i) &\geq \tau - (1+\alpha)m^i + u(x^i). \end{aligned} \tag{28}$$

Note that only the second row of (28), $\tau - n^i + u(x^i) \geq 0$, binds since the first one must be relaxed to maximize welfare by raising x^l and the third one cannot bind by definition of separating equilibrium. Similar to the previous cases, by plugging $\tau = (1-\frac{1}{\mu})\rho m^l + (1-\frac{1}{\eta})(1-\rho)n^i$ and the equilibrium conditions into it, we can have

$$\underbrace{\rho x^l\{u^{'}(x^l)-\frac{1}{\beta}\}}_{\text{Seigniorage from legal traders}} + \underbrace{(1-\rho)x^i\{u^{'}(x^i)-\frac{1}{\beta}\}}_{\text{Seigniorage from illegal traders}} + \underbrace{u(x^i)-x^iu^{'}(x^i)}_{\text{Trading gain of illegal traders}} \geq 0. \tag{29}$$

This incentive constraint implies that the sum of the seigniorage from legal traders and illegal traders and the trading gain of an individual illegal trader must be positive. Note that if $\mu = \eta$ then $x^l = x^i$ in equilibrium and this tax incentive constraint is equivalent to (25). Thus, we can have the same lower bound $\tilde{\eta} = \tilde{\mu} < 1$, at which $x^l = x^i = \tilde{x}$ satisfies this tax incentive constraint with equality.

Now we can consider whether we can improve welfare in a neighborhood of the allocation, $x^l = x^i = \tilde{x}$. By assuming $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$, we can transform the tax incentive constraint (29) into $\rho x^l\{u^{'}(x^l) - \frac{1}{\beta}\} + (1-\rho+\rho\gamma)u(x^i) - (1-\rho)\frac{1}{\beta}x^i \geq 0$. Notice that each function, $\rho x^l\{u^{'}(x^l) - \frac{1}{\beta}\}$ and $(1-\rho+\rho\gamma)u(x^i) - (1-\rho)\frac{1}{\beta}x^i$, is described as a Laffer curve which begins with zero. Since $\tilde{x}u^{'}(\tilde{x}) - \frac{1}{\beta}\tilde{x} < 0$ holds, $x^l\{u^{'}(x^l) - \frac{1}{\beta}\}$ is strictly decreasing in x^l in a neighborhood of $x^l = \tilde{x}$. Therefore, the slope of $\rho x^l\{u^{'}(x^l) - \frac{1}{\beta}\}$ at $x^l = \tilde{x}$ must be negative, i.e. $\rho(1-\gamma)u^{'}(\tilde{x}) - \rho\frac{1}{\beta} < 0$. In case of the function $(1-\rho+\rho\gamma)u(x^i) - (1-\rho)\frac{1}{\beta}x^i$, the slope at $x^i = \tilde{x}$, $u^{'}(\tilde{x}) - \frac{1}{\beta} + \{\rho(1-\gamma)u^{'}(\tilde{x}) - \rho\frac{1}{\beta}\}$, is also negative because of $u^{'}(\tilde{x}) - \frac{1}{\beta} < 0$ and $\rho(1-\gamma)u^{'}(\tilde{x}) - \rho\frac{1}{\beta} < 0$. Therefore, we can improve welfare at $x^l = x^i = \tilde{x}$ by raising η and reducing μ to decrease x^i and increase x^l as long as the tax incentive constraint is satisfied.

Let us briefly discuss the results in this subsection. In pooling equilibrium when β is sufficiently small, the tax incentive constraint could bind: Agents prefer current consumption to future consumption, so they avoid paying a lump-sum tax even though it improves the welfare of the whole society. This binding incentive constraint can be relaxed when we introduce dual currencies and separate the types. Given the binding constraint, we can increase the consumption of the legal traders by raising the rate of return on coins. It will reduce seigniorage from the legal traders, but we can offset this seigniorage loss by reducing the rate of return on paper money, because the increase in the seigniorage from the illegal traders dominates the decrease in the trading gain of an illegal trader. Thus, by transferring seigniorage implicitly, we can redistribute the trade surplus between the two types of agents as shown in the section 4.

6 Conclusion

We show that eliminating a money preferred by illegal traders can be suboptimal when government finance is restricted. In particular, our model shows that if illegal trade reduces social welfare, then it is optimal to tax illegal traders to finance a transfer to legal traders. This is a standard solution to an externality problem. In our model, this is achieved by using monetary policy to create an equilibrium in which the choice of currency is dependent on one's type. By doing so, policy can generate seigniorage from illegal traders to finance a transfer to legal traders. Nonetheless, our paper should not be the final word on the topic. One opportunity for future research would be to examine the issue of tax avoidance, such as whether tax revenue would increase sufficiently to replace lost seigniorage if cash was eliminated.

Additionally, the results of our paper show that exchange rates or transaction costs between different payment methods can be an alternative policy tool in the presence of heterogeneous preferences over alternative transaction methods. This might be related to a question raised by Stiglitz (2017) that asks whether we can control aggregate demand by adjusting the cost of different payment methods rather than changing the overall price level. Likewise, Agarwal and Kimball (2017) suggest that negative interest rates could be feasible by adjusting exchange rate between two payment methods, e-money and currency.[7] Work on this exchange rate channel could be also worthwhile in the future.

[7]Instead of implementing -2% in reserves, i.e. e-money in the paper, reducing the exchange rate of currency by 2% continuously, like $1, 0.98, 0.96, \ldots$, can satisfy no-arbitrage condition between currency and e-money. This mechanism is similar to our separating equilibrium case in which exchange rates grows at a fixed rate, but in our separating equilibrium the no-arbitrage condition does not hold because we have a heterogeneity in types.

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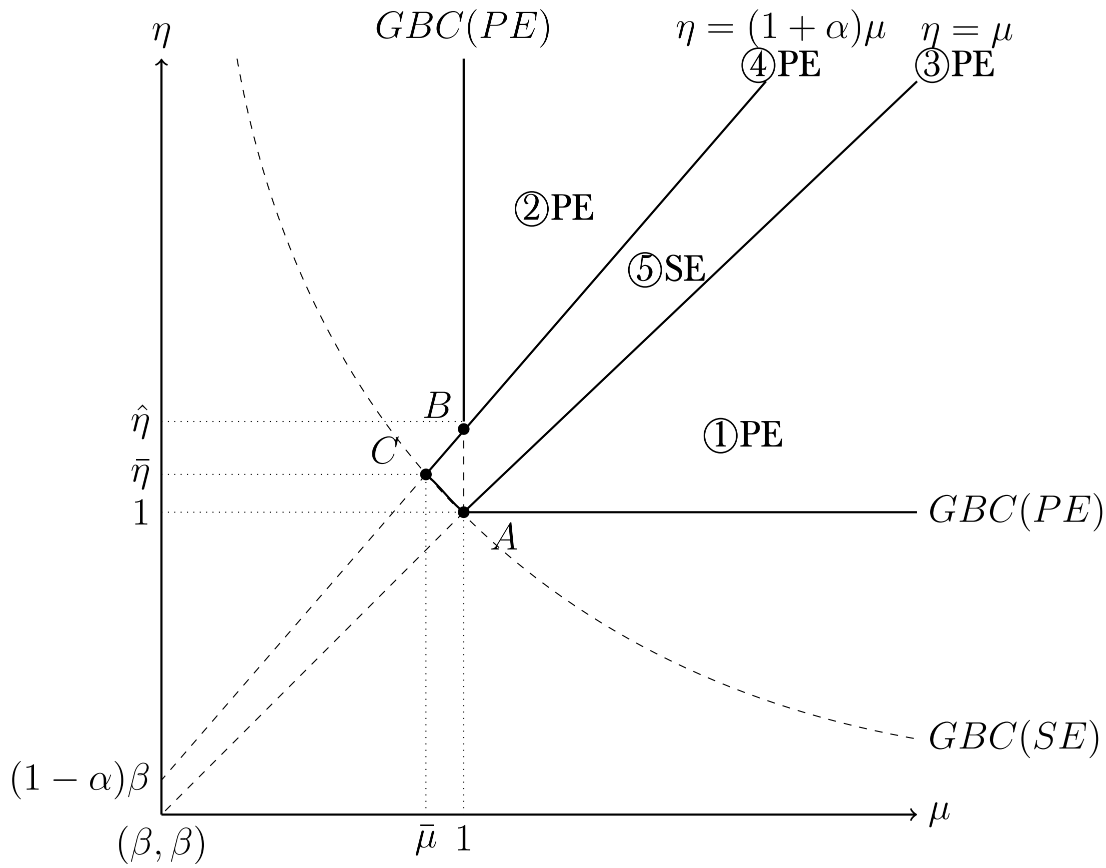

Figure 1: Types of Equilibria

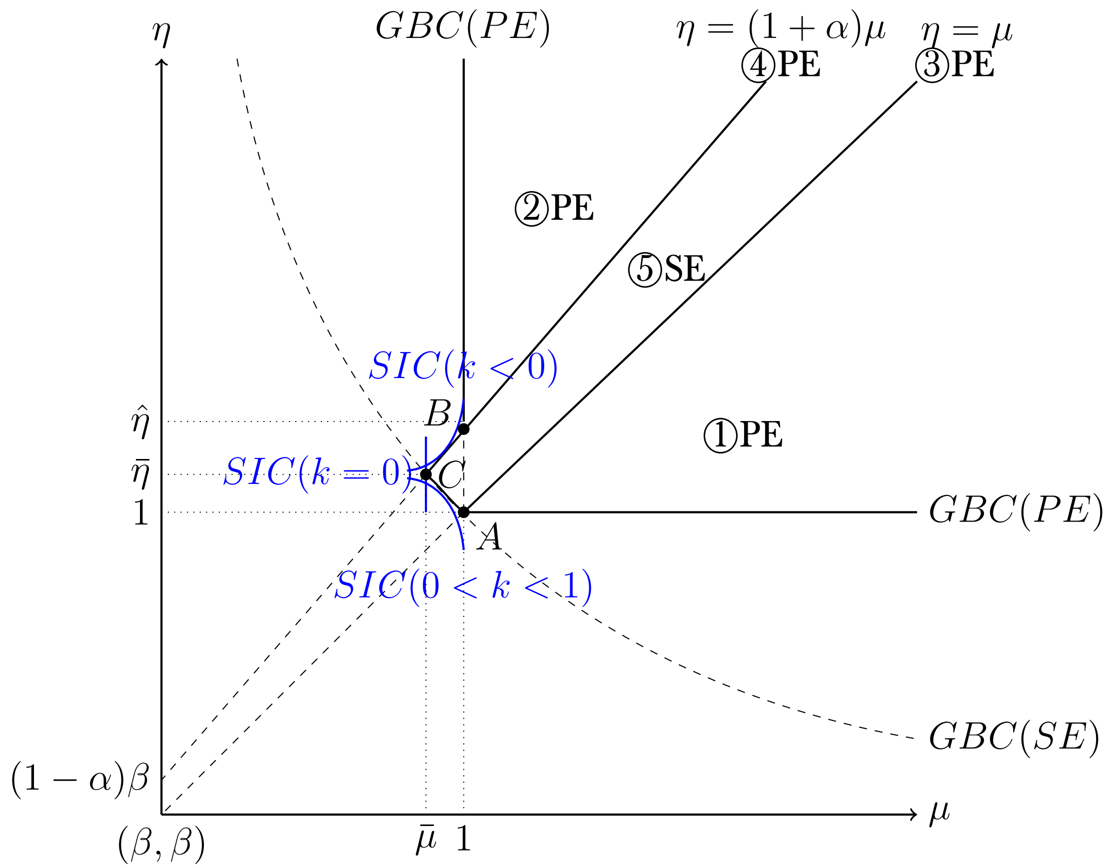

Figure 2: Welfare Analysis

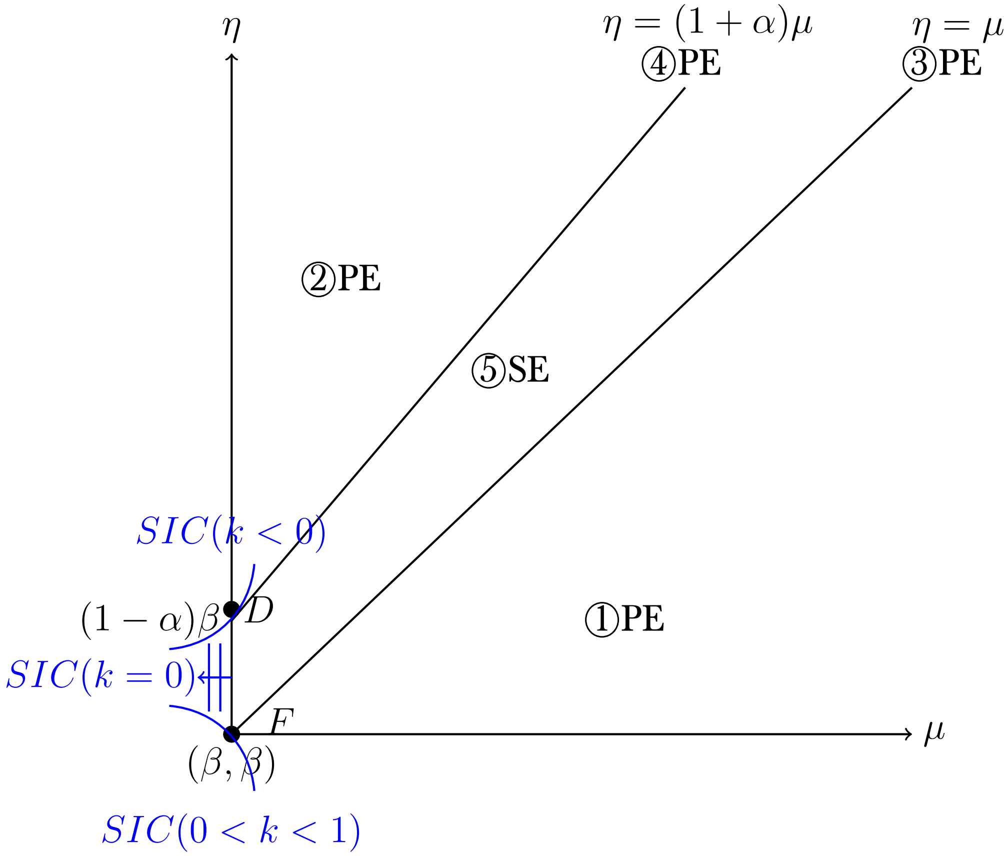

Figure 3: Without GBC